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Lesson No.

SECTION-A

SECTION-B

Department website : www.pbidde.org

LESSON NO. 1.1 **AUTHOR : DR. PARMOD KUMAR AGGARWAL**

INVENTORY CONTROL-I

1.1.0 OBJECTIVE OF THE LESSON

The objectives of the lesson is to study meaning, different types of inventories control in order to select optimum or economic order.

1.1.1 INTRODUCTION

Inventory is defined as any idle resource of an entreprise. It is a stock of goods, commodities or economic resources that are kept for the purpose of future business affairs. Inventory may be kept in any form—raw materials in process, finished product, packaging, spare and others—stocked in order to meet the expected demand. It is important to maintain some inventories for the smooth functioning of an enterprise. Suppose a firm is not maintaining inventories; if a sales order comes, it has to purchase the raw materials required, wait till these arrive and then start production. This increase the waiting time of customers to get the delivery goods.

1.1.2 MEANING OF INVENTORY CONTROL

Inventory control may be defined as ‘the function of directing the movement of goods through the entire manufacturing cycle from the conversion of raw materials to the inventory of finished goods in an orderly manner to meet the objectives of maximum customer-service with minimum investment and low-cost plant operation.

There are two basic functions.

(i) Maintaining an accounting record to handle the inventory transactions concerning each inventory item. For inventory transactions, a record-keeping system called *kardex file* is maintained for each inventory item.

(ii) Deciding inventory replenishment decisions. There are two basic replenishment decisions :

(a) When is it necessary to place an order (or produce) to replenish inventory? If the demand of an item is independent of that of other items, then the recorder point technique can be used to know the time of replenishment.

(b) How much is to be ordered (or produced) in each replenishment?

The decision about the number of units to order (or produce) for replenishment depends on the types of inventory costs.

1.1.3 REASONS FOR CARRYING INVENTORIES

It is essential for any firm to have inventory because of the following reasons :

1. It provides adequate service to customers.
2. It helps in smooth and efficient running of business.
3. It reduces the possibility of duplicating orders.
4. Timely shipment of customers' order will improve cash flow.
5. It takes care of economic fluctuations.
6. It helps in minimising the loss due to deterioration, obsolescence, damage and, so on.
7. It acts as a buffer stock when raw materials are received late and shop rejections are too many.
8. It takes advantages of price discounts by bulk purchasing.
9. It improves the manpower, equipment and facility utilisation because of better planning and scheduling.

1.1.4 TYPES OF INVENTORIES

Basically, there are five types of inventories.

(i) **Fluctuation Inventories (Buffer Inventories) :** These are inventories to meet uncertainties of demand and supply. Buffer inventories in excess of those necessary to meet the average demand during lead time (the time lapsing between placing an order and having the goods in stock ready for use) and held for protecting against the fluctuations in demand and lead time, are termed as safety stocks or reserve stocks.

(ii) **Anticipation Inventories :** These are built up in advance for the season of large sales, a promotion programme, or a plant shut down period. It keeps men and machine ready for future needs. For example, keeping crackers well before Diwali or air coolers or air conditioners before summer.

(iii) **Cycle (Lot Size) Inventories :** These are built up in advance because the purchases are usually made in lots rather than for the exact amounts needed at a point of time.

(iv) **Transportation Inventories (or Pipeline Inventories) :** Such inventories exist because of the transportation of inventory items to various distribution centres and customers from the production centres. This type of inventory is also called *process inventory* where the significant amount of time is consumed in the transshipment of items from one location to another. To meet the demand it is essential to hold extra stock at various work stations.

(v) **Decoupling Inventories :** If various production stages operate successively, then the breakdown of one or many may affect the entire system. This kind of interdependence is not only expensive but also disruptive for the entire system. The inventories used to reduce the interdependence of various stages of the production system are known as decoupling inventories. These inventories may be classified as :

(a) **Raw materials and component parts :** It is used to decouple the producer from the suppliers. That is, raw materials and component parts inventory could
  - (i) act as a buffer to take care of delays on the part of suppliers.
  - (ii) guard against seasonal variations in the demand of final product.

(b) **Work-in-process inventory :** As it takes time to convert raw material into finished product, work-in-process inventory is developed. This inventory takes the form of orders waiting to be transported between machines or of orders waiting to be processed on a particular machine. The level of such inventory can be increased by changing the production process, lot sizes, and so on.

(c) **Finished good inventory :** It is the inventory of final products which could be released for sale to the customers. The size of this inventory depends upon the demand and the ability of the firm to sell its products.

(d) **Spare parts inventory :** There are the parts which are used in the production process but do not become part of the product.

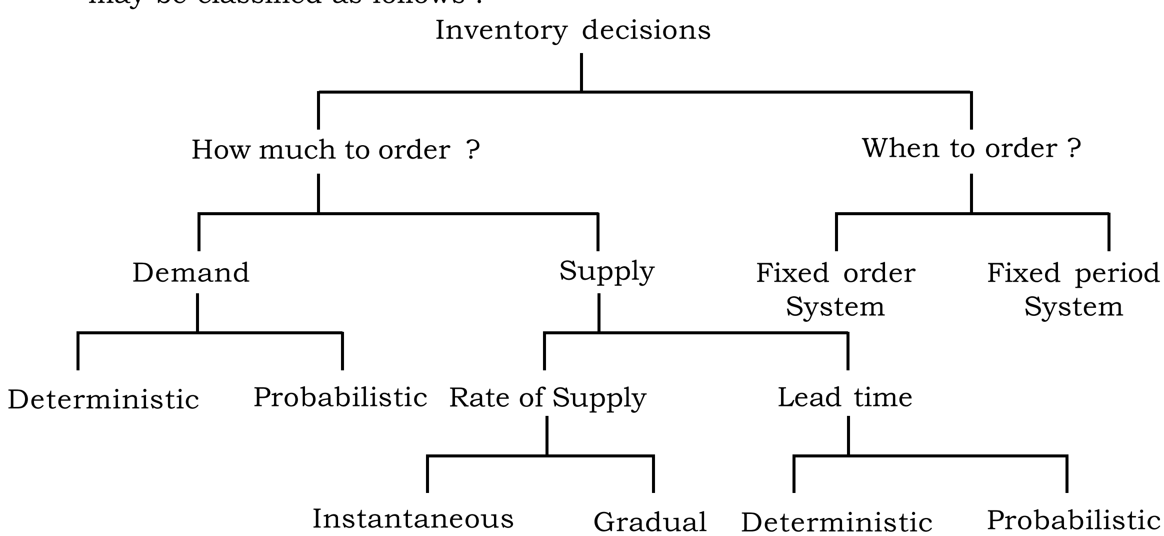
Operating Constraints : The stock level of various items in the inventory is governed by various constraints such as limited warehouse space, limited budget available for inventory, customer service level to be achieved, and management attitude about the individual items in the inventory.

1.1.5 OPERATING DECISION RULES

Two types of decision need to be made in managing inventories :

(i) How much (size) is to order when the inventory of that item is to be replenished ?

(ii) When to place an order (or set up production) to replenish inventory? These may be classified as follows :



1.1.6 COSTS INVOLVED IN INVENTORY PROBLEMS

Various costs involved in inventory problems may be generally classified as follows :

1. Set-up Cost/Orderting Cost (C_o) : These include the fixed cost associated with obtaining goods through placing in orders or purchasing or manufacturing or setting up a machinery before starting production. The costs include ordering of raw materials for production purposes, advertisements, postage, telephone charges, travel expenditure and so on. These are also called order costs or replenishment costs per production run. These are assumed to be independent of the quantity ordered or produced. Ordering cost may be calculated as :

Order cost = (Cost per order) × (Number of orders)

2. Purchase or Production Cost : The cost of purchasing (or producing) a unit of item is known as purchase (or production) cost. Purchase cost per unit item is affected by the quantity purchased due to *quantity discounts* or *price breaks.*

Purchase cost = (Price per unit item) × (Demand per unit item)

When price break or quantity discounts are available for bulk purchase of a specified quantity, the unit price becomes smaller as size of order Q exceeds a specified quantity level. In such cases the purchase cost becomes variable and depends on the size of the order. In this purchase cost is given by

$$\text{Purchase cost} = \begin{pmatrix}\text{Price per unit when} \\ \text{order size is Q}\end{pmatrix} \times \begin{pmatrix}\text{Demand per} \\ \text{unit time}\end{pmatrix}$$

3. Carrying or Holding Cost (C_c) : The cost associated with carrying or holding goods in stock is called holding or carrying cost per unit of item for a unit of time. It is assumed to vary directly with the size of inventory as well as the time the item is held in stock. This cost generally includes :

(i) *Invested Capital Cost :* This is the interest charged on the capital investment.

(ii) *Record-keeping and Administrative Cost :* This shows that there is a need of funds for maintaining the records and necessary administration.

(iii) *Handling Costs :* These include all costs associated with movement of stock, cost of labour, and so on.

(iv) *Storage Costs :* These involve the rent for storage space or depreciation and interest even if own space is used.

(v) *Depreciation, Deterioration, Obsolescence Costs :* These costs arise due to the items in stock being out of fashion or the items undergoing chemical changes during storage, date expiring and so on.

(vi) *Taxes and Insurance Costs :* These costs require careful study and generally amounts to 1 percent to 2 percent of the invested capital.

4. Shortage (or Stock out) Cost : The shortage of items occurs when actual demand cannot be fulfilled from the existing stock. These costs arise due to shortage of goods, and may cause loss of sales. Goodwill may be lost either by a delay in meeting the demand or being unable to meet the demand. The shortage can be looked at in two different ways :

(i) The supply of items is awaited by the customers, that is, the items are back ordered and therefore there is no loss in sale.

(ii) Customers are not ready to wait and, therefore, there, is loss of sale. In the case of shortage, cost shall be measured in terms of goodwill lost and lost profit on the unit which were demanded but were not available.
Shortage cost may be calculated as :

$$\text{Shortage cost} = \begin{pmatrix}\text{Cost of being one unit} \\ \text{short in the inventory} \\ \text{planning period}\end{pmatrix} \times \begin{pmatrix}\text{Average number of units} \\ \text{short in the inventory} \\ \text{planning period}\end{pmatrix}$$

Average number of units short =

$$\left(\frac{\begin{pmatrix}\text{Minimum shortage in the} \\ \text{inventory planning period}\end{pmatrix} + \begin{pmatrix}\text{Maximum shortage in the} \\ \text{inventory planning period}\end{pmatrix}}{2}\right) \times \begin{pmatrix}\text{Time for which} \\ \text{Shortage occurs}\end{pmatrix}$$

5. Salvage Cost (or Selling Price) : When the demand for commodity is affected by the quantity stocked, the decision model of the problem depends upon the profit maximisation criterion and includes revenue from selling. Generally, salvage value may be combined with the cost of storage and not considered independently.

6. Revenue Cost : When it is assumed that both the price and the demand of the product are not under the control of the organisation, the revenue from the sales is independent of the company's inventory policy. It may be neglected expect when the organisation cannot meet the demand and the sale is lost. the revenue cost may or may not be included in the study of inventory.

7. Total Inventory Cost : If the unit price of an item depends on the quantity purchased, that is, price discount is available, then we formulate an inventory policy that considers the purchase cost of the items held in stock. The total inventory cost is given as :

Total inventory cost = Ordering cost + Carrying cost + Shortage cost

8. Total Variable Inventory Cost : When price discounts are not offered, the purchase cost remains constant and is independent of the quantity purchased. Hence, $TVC = C_O + C_C + S_C$

1.1.7 OTHER FACTORS IN INVENTORY ANALYSIS

The factors which play an important role in the study of inventory problems are :

1. Demand : Demand is the number of units required per period and may be known exactly or in terms of probabilities or be completely unknown. If demand is known, it may be either fixed or variable per unit time. Problems in which the demand is known and fixed are called *deterministic* problems. If the demand is assumed to be a random variable, then those problems are called *stochastic* or *probabilistic problems.*

2. Lead Time : The time gap between placing of an order and its actual arrival in the inventory is known as lead time. The Longer the lead time, the higher is the average inventory. Lead time has two components, namely *administrative lead time*—time from the initiation of procurement action to the placing of an order, and *delivery lead time*—time from placing of an order to the delivery of the ordered material.

3. Order Cycle : The time period between placement of two successive orders is referred to as an order cycle. The order may be placed on the basis of following two types of inventory review systems.

(a) *Continuous review :* The record of the inventory level is checked continuously until a specified point is reached where a new order is placed.

(b) *Periodic review :* The inventory levels are viewed at equal time intervals and orders are placed at such intervals. The quantity ordered each time depends on the available inventory level at the time of review.

4. Stock Replenishment : Actually, the replenishment of stock may occur instantaneously or uniformly. Instantaneous replenishment occurs in case the stock is purchased from outside sources whereas uniform replenishment occurs when the product is manufactured by the company.

5. Time Horizon : The time period over which the inventory level will be controlled is called the time horizon.

6. Reorder Level : The level between maximum and minimum stock, at which purchasing (or production) activities start for replenishment is called *reorder level.*

Variables in Inventory Problem : The variables used in any inventory model are of two types :

(a) Controlled variables

(b) Uncontrolled variables

(a) *Controlled variables :* The following variables are controlled separately or in combination.
   (i) How much quantity to buy (purchase, production, so on)
   (ii) The frequency or timing of acquisition—how often or when to replenish the inventory?
   (iii) The completion stage of stocked items.

(b) *Uncontrolled Variables :* These include the holding costs, shortage or penalty costs, set up costs, demand, lead time, and supply of goods.

1.1.8 EOQ MODELS

1.1.8.1 Economic Order Quantity (EOQ) - Meaning

The concept of economic ordering quantity was first developed by F. Harries in 1916. The inventory problems in which the demand is assumed to be fixed and completely predetermined as usually referred to as economic order quantity (EOQ). When the size of order increases, the ordering costs (cost of purchase, etc.) will decrease, whereas the carrying charges (cost of storage, insurance etc.) will increase. Hence, there are two opposite costs in the production process, one encourages increase in the order size and the other discourages.

1.1.8.2 Determination of EOQ by Tabular Method

The method involves the following steps :

Step 1 : Select a number of possible lot sizes of purchase.

Step 2 : Determination of total cost for each lot size selected.

Step 3 : Finally, select the ordering quantity which minimises total cost.

Exampl : A stockist purchases an item at the rate of Rs. 4 per piece from a manufacturer. 2000 units of the item are required per year. What should be the order quantity per order if the cost per order is Rs. 32 and the inventory charges per year are 20 paise and manufacturer insists that the item must be purchased in the units of 100.

Solution : Here, demand\year = 2000 units

Ordering Cost = Rs. 32 per order

Inventory carrying cost = 0.20 x 4 = Rs. 0.8\unit\year

Annual ordering cost = ordering cost x Total number of orders

Annual carrying cost = carrying cost\unit x Average inventory carried

We apply this concept in the Table below where start with the order size of 100 units and its increments of 100 units till we get the optimal order size.

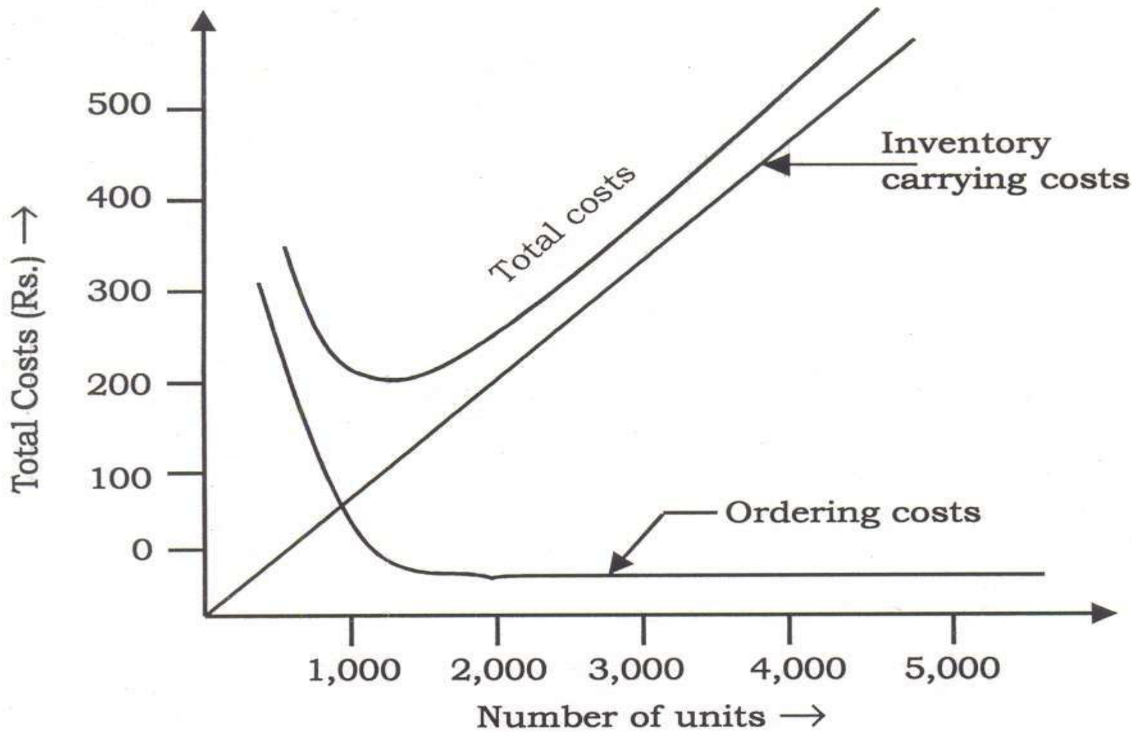
Table : Determination of EOQ

Order Size	No. of orders per year	Annual Ordering Cost (Rs.)	Annual Carrying Cost (Rs.)	Total Cost\Year (Rs.)
100	20	32x20=640	$0.8x\frac{100}{2} = 40$	680
200	10	320	$0.8x\frac{200}{2} = 80$	400
300	$\frac{2000}{300}$	$\frac{20x32}{3} = \frac{640}{3}$	0.8x150 = 120	$\frac{1000}{3} = 333.33$
400	5	5x32=160	0.8x200 = 160	320 *Optimal
500	4	4x32=128	0.8x250 = 200	328
600	$\frac{10}{3}$	$\frac{320}{3}$	0.8x300 = 240	$\frac{1040}{3} = 346.6$

As soon as the total cost starts increasing, we stop at the next step. Thus for the optimal cost Rs. 320, the optimal order size is 400 units.

1.1.8.3 Determination of EOQ by Graphical Method

The data calculated in tabular method can be graphed as below :



The minimum total cost occurs at the point where the ordering costs and inventory carrying costs are equal. A disadvantage of the graphical method is that without specific costs and values, an accurate plotting of the carrying costs, ordering costs, and total costs is not feasible.

1.1.8.4 Determination of EOQ by Algebraic Method

It is based on following assumptions :

(i) Demand is known and uniform

(ii) Lead time is zero

(iii) Production rate is infinite i.e. production is instantaneous

(iv) Shortages are not allowed

(v) Set-up cost per production is given

(vi) Holding cost/Carrying cost can be expressed as a percentage of the value of average inventory.

This method overcomes the problems of tabular and graphic method and is widely used. It is based on the fact that total cost would be minimum when carrying costs and ordering costs are equal. The step by step procedure to determine EOQ can written as :

(i) Annual ordering cost (C_O)

$$= \text{No. of orders place/year} \times \text{Ordering cost period}$$

$$= \frac{\text{Annual demand}}{\text{No. of units in each order}} \times \text{Ordering cost per period}$$

$$= \frac{DC_O}{Q}$$

(ii) Annual Carrying or holding cost =

= Average Inventory Level × Carrying Cost/Unit/Year

$$= \frac{\text{Ordered Quantity}}{2} \times \text{Carrying cost/unit/year}$$

$$= \frac{Q}{2}.C_O$$

(iii) Economic/Optimal Order Quantity is determined when ordering cost = Carrying Cost

i.e. $\frac{DC_O}{Q} = \frac{QC_C}{2}$

$$2\ DC_o = Q^2.C_C$$

$$Q^2 = \frac{2DC_O}{C_C}$$

$$Q = \sqrt{\frac{2DC_O}{C_C}}$$

(iv) Total inventory cost or Total variable cost

$$= \frac{DC_O}{Q} + \frac{QC_C}{2}$$

$$TVC = \sqrt{2DC_O.C_C}$$

(v) Number of orders (N) = $\frac{\text{Total demand}}{\text{EOQ}}$

(vi) Procurement Period/Time between Orders = $\frac{\text{EOQ}}{\text{Demand per annum}}$

(vii) Ordering Cycle = $\frac{\text{Time in no. of days / months / years}}{\text{Number of orders}}$

For example : A company purchases raw material from outside suppliers for annual requirement. During the coming year, the company plans to manufacture at a constant 1,00,000 units of its products. The cost of placing each order is Rs. 160 for any item in inventory, the company incurs an carrying cost annually equal to 20% of the item cost. Find the following :

(i) What is EOQ?

(ii) What is the total inventory cost?

(iii) How many orders will be placed in next year?

Solution.

C_O = 160; C_O = Ordering Cost; C_{PU} = Cost per unit = Rs. 20;

C_C = 20% of inventory item; C_C = Carrying cost

$= \frac{20}{100} \times 20$ = Rs. 4 per unit annum

(i) EOQ = $\frac{2DC_O}{C_C} = \sqrt{\frac{20 \times 100 \times 160}{4}}$ = 282.8 units.

(ii) Total inventory cost

= Material Cost + Total Material cost

$= D \times C_{PU} + \sqrt{2DC_OC_C}$

$= 100000 \times 20 + \sqrt{2 \times 100000 \times 4 \times 20}$

= 200000 0+ 11314 = Rs. 2011314

(iii) Number of Orders (N)

$= \frac{D}{Q}$

$= \frac{100,000}{2828}$ = 353.36 orders

= 354 Orders.

Note : Next method, References and Exercise are given in Lesson number 12.

LESSON NO. 1.2 **AUTHOR : DR. PARMOD KUMAR AGGARWAL**

INVENTORY CONTROL-II

1.2.0 OBJECTIVE OF THE LESSON

In this lesson, different models with different methods are discussed with examples.

1.2.1 ECONOMIC LOT SIZE MODEL WHEN DEMAND IS UNIFORM AND SHORTAGES ARE PERMITTED :

When demand or consumption is in excess of supply or production, it will result in shortages. Such situation affect the goodwill of concern and may lead to permanent decline in sales and profit. In this case following formulas can be used :

(i) $\text{EOQ} = \sqrt{\dfrac{2DC_O}{C_C}\cdot\left(\dfrac{C_S + C_C}{C_S}\right)}$

C_S = Shortage Cost, D = Annual Demand
C_O = Ordering Cost and C_c = Carrying Cost

(ii) Back order units or shortage quantity (S) = EOQ × $\dfrac{C_C}{C_S + C_C}$

(iii) Total Shortage Cost = S × C_S

(iv) Total Variable Cost (TVC) = $\sqrt{2DC_OC_C\left(\dfrac{C_S}{C_S + C_C}\right)}$

(v) Maximum Inventory Level = EOQ – S

For Example : Oswal Sugar Ltd. has demand for 18,000 tonnes sugarcane per year. The sugarcane price is Rs. 1200 per tonne. The holding cost is 0.1 percent of sugarcane price and cost of shortage is Rs. 5 per tonne. The production set-up is Rs. 400. calculate EOQ and its parameters.

Solution. Demand (D) = 18,000 tonne

Inventory price C_{PU} = 1200 per tonne

Carrying cost C_C = 0.1 % of 1200

$$= \frac{1}{1000} \times 1200 = 1.2$$

Shortage cost C_S = Rs. 5 per tonne

$$\text{EOQ (Q)} = \sqrt{\frac{2DC_O}{C_C} . \left(\frac{C_S + C_C}{C_S}\right)}$$

$$= \sqrt{\frac{2 \times 18000 \times 400}{1.20} \times \frac{1.20 + 5}{5}}$$

$$= \sqrt{14880{,}000} = 3857 \text{ tonnes}$$

$$\text{Shortage quantity (S)} = \text{EOQ} \times \frac{C_S}{C_C + C_S}$$

$$= 3857 \times \frac{1.20}{1.20 + 5}$$

$$= 3857 \times \frac{1.20}{6.20} = 746 \text{ tonnes}$$

Shortage Cost = S × C_S = 746 × 5 = 3730 Rs.

$$\text{TVC} = \sqrt{2DC_O . C_C \left(\frac{C_S}{C_S + C_C}\right)}$$

$$= 2 \times 18000 \times 400 \times 1.20 \left(\frac{5}{5 + 1.2}\right)$$

= Rs. 20785

1.2.2 EOQ PRODUCTION MODEL WHEN SHORTAGES ARE NOT PERMITTED :

In this model, supply is finite i.e. inventory is not replenished immediately rather it builds up over time period because production rate (r) is greater than demand rate (D) i.e. r > D. It is based on following assumptions.

(a) Demand is exactly known and uniform

(b) Shortages are not permitted

(c) Inventory is building up at a constant (r – D) units per annum (t_1)

(d) There is no replenishment during time t_2 following formulas are used to determine various parameters :

(i) $$\text{EOQ} = \sqrt{\frac{2DC_O}{C_C} \left(\frac{r}{r - D}\right)}$$

Where D = annual demand/consumption rate

r = Production rate

(ii) TC (Total Cost) = $\sqrt{2DC_O.C_C\left(\frac{r-D}{r}\right)}$

(iii) Usage Cycle/Consumption Cycle = $\frac{EOQ}{D \text{ per day}}$

(iv) Production Cycle = $\frac{EOQ}{r \text{ per day}}$

(v) Number of production run (N) = $\frac{D}{EOQ}$

For example : Using following information,

Production rate (r) = 200,000 Units per year

Demand rate (D) = 20,000 Units per year

Set up Cost (C_O) = Rs. 60

Holding Cost (C_C) = Rs. 2 per unit per year = $\frac{2}{100} \times 1 = 0.02$

Inventory price (C_{PU}) = Rs. 1

Calculate : EOQ, production cycle, number of production runs, usage cycle and TC.

Solution.

$$EOQ = Q = \sqrt{\frac{2DC_O}{C_C}\left(\frac{r}{r-D}\right)}$$

$$= \sqrt{\frac{2 \times 20{,}000 \times 60}{0.02} \times \frac{200{,}000}{200{,}000 - 20000}}$$

$$Q = \sqrt{13333333.3} = 3651 \text{ Units}$$

$$\text{Production Cycle} = \frac{EOQ}{r \text{ per day}} = \frac{3651}{200000/365} = 6.6 \text{ days} = 7 \text{ days}$$

$$\text{Number of production runs} = \frac{D}{EOQ}$$

$$= \frac{20{,}000}{3651} = 5.48 \text{ production runs per year}$$

$$\text{Usage Cycle} = \frac{EOQ}{D \text{ per day}}$$

$$= \frac{3651}{20{,}000/365} = 66.63 \text{ days} = 67 \text{ days}$$

$$TC = \sqrt{2DC_O.C_C\left(\frac{r}{r-D}\right)}$$

$$= \sqrt{2 \times 20{,}000 \times 60 \times 0.02 \times \frac{200{,}000}{200{,}000 - 20000}}$$

$$= \text{Rs. } 730$$

1.2.3 EOQ PRODUCTION MODEL WHEN SHORTAGES ARE PERMITTED :

This model is also based on same assumptions as previous model. Following formulas are used to determine various parameters of the model :

(a) $$EOQ = Q = \sqrt{\frac{2DC_O}{C_C}\left(\frac{r}{r-D}\right)\left(\frac{C_S + C_C}{C_S}\right)}$$

Where D = annual demand/consumption rate
r = Production rate

(b) $$\text{Shortage quantity (S)} = EOQ.\frac{r-D}{r}.\frac{C_C}{C_C + C_S}$$

(c) $$\text{Usage Cycle/Time between orders} = \frac{EOQ}{\text{Demand per day}}$$

(d) $$\text{Manufacturing Time} = \frac{EOQ}{\text{production per day}}$$

(e) $$TC = \sqrt{2DC_O.C_C\left(\frac{r-D}{r}\right).\left(\frac{C_S}{C_C + C_S}\right)}$$

(f) $$\text{Maximum inventory Level} = \left(\frac{r-D}{r}\right).(EOQ - S)$$

For example : For the following data :

Demand per year (D) = 35000 Units
Production rate (r) = 45000 Units per year
Inventory price (C_{PU}) = Rs. 75 per unit
Ordering Cost (C_O) = Rs. 400 per order
Carrying Cost (C_C) = Rs. 25 per unit per year
Shortage Cost/Back order Cost (C_S) = Rs. 65 per unit
Number of working days in a year = 300 days

Calculate (i) EOQ
(ii) Back Order Units
(iii) Manufacturing time
(iv) Consumption time

(v) Total cost

(vi) Maximum inventory Level

Solution. (i)

$$EOQ = Q = \sqrt{\frac{2DC_O}{C_C}\left(\frac{r}{r-D}\right)\left(\frac{C_S+C_C}{C_S}\right)}$$

$$= \sqrt{\frac{2\times 35000\times 400}{25}\times\frac{45000}{45000-35000}\times\frac{65+25}{65}}$$

= 2642 units

(ii)

$$\text{Back Order Units (S)} = EOQ\left(\frac{r-D}{r}\right)\left(\frac{C_C}{C_C+C_S}\right)$$

$$= 2642\times\frac{45000-35000}{45000}\times\frac{25+65}{65}$$

= 163 units

(iii)

$$\text{Manufacturing time} = \frac{EOQ}{r \text{ per day}}$$

$$= \frac{2642}{45000/300} = 17.61 = 18 \text{ days}$$

(iv)

$$\text{Consumption time} = \frac{EOQ}{D \text{ per day}}$$

$$= \frac{2642}{35000/300} = 22.65 = 23 \text{ days}$$

(v)

$$TC = \sqrt{2DC_O.C_C\left(\frac{r-D}{r}\right)\left(\frac{C_S}{C_S+C_C}\right)}$$

$$= \sqrt{2\times 35000\times 400\times 25\times\frac{45000-35000}{45000}\times\frac{65}{65+25}}$$

= 10599 Rs.

(vi)

$$\text{Maximum inventory Level} = \left(\frac{r-D}{r}\right)(EOQ-S)$$

$$= \frac{45000-35000}{45000}(2642-163) = 423 \text{ units.}$$

1.2.4 PROBABILISTIC INVENTORY MODEL

So far we have discussed the inventory models where demand was known exactly. But in real world life, demand is seldom known with certainty. However, we determine the probability distribution of future demand with the help of statistical methods. This model is useful for perishable goods, seasonal items, fashion goods, demand of newspapers etc. In this model, we minimise the total expected costs rather than actual costs.

1.2.5 ASSUMPTIONS OF THE MODEL

(i) Lead time is assumed to be zero.

(ii) Production is instantaneous

(iii) C_1 is potential profit per unit. It may be called shortage cost in case supply is less than demand.

(iv) C_2 is the potential loss per unit, it may be called carrying cost or holding cost in case supply is more than demand.

Consider the case of a perishable item, say newspaper problem. Demand for today's newspaper is assumed to be zero on next day. In other words, unsold newpapers are of no use tomorrow. If supply of newspaper is more than demand, then newspaper seller has to suffer losses due to unsold newspapers. On other hand, if demand is more than he will have to suffer a loss of profit which he could not earn. So, the problem of newspaper seller is to decide the number of newspapers to be ordered every day to maximise his expected profits. In this type of problem, following steps are required :

(i) C_1 = Selling Price – Purchase price

(ii) C_2 = Purchase Price – Scrap value if any

(iii) Cost ratio can be defined as = $\frac{C_1}{C_1 + C_2}$

(iv) Determine the optimum quantity to be ordered by compairing cost ratio with commulative probability. Optimum quantity 'q' is obtained where the value of commulative probability exceeds the cost ratio i.e.

$$P\ (r \geq q - 1) < \frac{C_1}{C_1 + C_2} \leq P\ (r \leq q)$$

Example 1. The daily demand of a bread at a bakery follows a discrete distribution as follows :

S.No.	Demand	Probability
1.	25	0.2
2.	26	0.11
3.	27	0.10
4.	28	0.09
5.	29	0.08
6.	30	0.12
7.	31	0.14
8.	32	0.05
9.	33	0.04
10.	34	0.04
11.	35	0.03

The purchase price of a bread is Rs. 8 per packet. The selling price is Rs. 11 per packet. If the bread packets are not sold within a day of purchase, they are sold at Rs. 4 per packet to hotels for secondary use. Find the optimal size of the bread.

Solution. Purchase Price = Rs. 8

Selling price per packet = Rs. 11

Salvage price/packet = Rs. 4

Marginal cost of surplus C_2 = 8 – 4 = Rs. 4

Marginal cost of shortages C_1 = 11 – 8 = Rs. 3

$$\text{Commulative Probability } P = \frac{C_1}{C_1 + C_2}$$

$$= \frac{3}{3+4} = 0.43$$

Commulative Probability of Demand

S.No.	Demand	Probability	Commulative Probability
1.	25	0.2	0.2
2.	26	0.11	0.31
3.	27	0.10	0.41
4.	28	0.09	0.50
5.	29	0.08	0.58
6.	30	0.12	0.70
7.	31	0.14	0.84
8.	32	0.05	0.89
9.	33	0.04	0.93
10.	34	0.04	0.97
11.	35	0.03	1.00

from the table, we find that

$$P_3 < \frac{C_1}{C_1 + C_2} = 0.43 < P_4$$

i.e. 0.41 < 0.42 < 0.5

$\therefore$ Optimal size is D_4 which is equal to 28 breads.

1.2.6 REFERENCES

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2. Sharma, K.K. : *Quantitative Techniques and Operations Research*
3. Kapoor, V.K. : *Operations Research & Management Techniques.*

1.2.7 EXERCISE

1. Explain the following terms :
   (a) Set-up cost
   (b) Carrying cost

(c) EOQ
(d) Lead time
(e) Safety stock
(f) Maximum Inventory Level

2. An aircraft company uses rivets at an approximate rate of 25,000 kg. per year. The rivet cost Rs. 25 per kg. and inventory carrying cost is 1% per month. How frequently should orders for rivets be placed and what quantities should be ordered ?

3. A company uses a special bracket in the manufacturing of its product which it orders from outside suppliers. The appropriate data is :

   Demand = 2000 items per year
   Ordering cost = Rs. 20 per order
   Carrying cost = 20% of item price
   Basic item price = Rs. 10 per bracket

   The company is offered the following discounts on the basic price.
   (a) for order of quantities : 400-799 (2% discount)
   (b) for order of quantities : 800-1599 (4% discount)
   (c) for order of quantities : 1600 and above (5% discount)
   Determine the most economical quantity to order.

4. A newspaper boy buys papers for 4 paise each and sells them for 9 paise each. He cannot returns unsold newspapers. Daily demand has the following probability distribution :

Demand	**Probability**
20	0.01
21	0.02
22	0.07
23	0.09
24	0.18
25	0.15
26	0.25
27	0.15
28	0.03
29	0.05

If each day's demand is independent of the previous day, how many papers should he order each day ?

5. An electrical component manufacturing firm uses valves at the rate of 2500 valves per year. The cost of the single valve is Rs. 1 while the inventory carrying cost is 20% of the value of the stocks. The valves are supplied in lots and the cost of procurement, including trasnportation costs is Rs. 40 per order.

   Suppose that the actual consumption of this valve increases by 500 valves over the year against the estimated requirement of 2500 valves, determine the extent of loss of the firm will incur by using EOQ calculated for the usage of 2500 valves.

LESSON NO. 2.2 | **AUTHOR : P.C. GARG**

NETWORK ANALYSIS : INTRODUCTION TO PERT/CPM

2.2.0 OBJECTIVE OF THE LESSON :

Objective of the lesson is to tell how logical sequence of various activities to be performed to achieve various project objectives.

2.2.1 INTRODUCTION :

The Operations Research techniques used for planning, scheduling and controlling large and complex projects are often referred to as network analysis. A network is a graphical diagram consisting of certain configuration of arrows and nodes for showing the logical sequence of various activities to be performed to achieve the project objectives. Two most popular techniques used in many scheduling situations are the critical path method or simply CPM and the Programme Evaluation and Review Technique or popularly known as PERT.

2.2.2 BASIC STEPS IN PERT/CPM TECHNIQUES

The procedure involved in applying PERT/CPM consists of the four main steps.

Step 1. **Planning :-** The planning phase is started by splitting the total project into small projects. These small projects are divided into activities and are analysed by the department. The relationship of each activity with respect to other activities are defined.

Step 2. **Scheduling :-** The ultimate object of the scheduling phase is to prepare the time chart showing the start and finish times for each activity as well as its relationship to other activities of the project.

Step 3. **Allocation of Resources :-** The allocation of resources is performed to achieve the desired objective. A resource is a physical variable such as labour, finance, equipment and space.

Step 4. **Controlling :-** The final phase in project management is controlling. CPM facilitates the application of the principle of management in identifying the areas that are critical to the completion of the project. By having progress report from time to time and updating the network continuously, a better financial as well as technical control over the project is exercised.

2.2.3 BASIC CONCEPTS OF NETWORK ANALYSIS

In project scheduling, the first step is to sketch an arrow diagram which shows the interdependence and precedence relationship between activities of the project. In a network representation, certain basic definitions are used.

(i) **Activity :-** Any individual operation which utilizes resources and has an end and a beginning is called an activity. An arrow is commonly used to represent an activity with its head indicating the direction of progress of the project.

(ii) **Dummy Activity :-** Any activity which does not consume any kind of resource but merely depicts the technological dependence is called a dummy activity.

(iii) **Event :-** An event represents a point in time signifying the completion of some activities and the beginning of new ones. This is usually represented by a circle in a network which is also called a node or connector.

(iv) The first prerequisite in the development of a network is to maintain the precedence relations. In order to make a network, following points should be kept into consideration.

(a) What job or jobs precede it ?

(b) What job or jobs could run concurrently ?

(c) What job or jobs succeed it ?

(d) What controls the start and finish of a job ?

(v) **Labelling :-** For the network, it is necessary that various nodes are properly labelled. A standard procedure called the I-J rule developed by Fulkerson is most commonly used for this purpose. Main steps of this procedure are :

(a) A start event is one which has arrows emerging from it but none entering it. Find the start event and number it as unity.[1]

(b) Number all the remaining events as 2, 3 and so on from top to bottom and left to right that may facilitate other users in reading the network diagram.

(c) Go on repeating the step (b) Until the end is reached.

2.2.4 TIME ESTIMATES IN NETWORKS

For each activity an estimate must be made of time that will be spent in the actual accomplishment of that activity. The next step after making the time estimates is the calculation of earliest times and latest times for each node. These calculations are done in the following way.

(a) Let zero be the starting time for the project. Then for each activity there is an earliest starting time (ES) relative to the project starting time, which is the earliest possible time when an activity can begin, assuming that all of the predecessors also are started at their ES. Then for that activity, its earliest finish time (EF) is simply the ES + activity time.

Thus, if ESi denotes the earliest start time of all the activities emanating from event i and tij is the estimated time of activity (i, j), then

EFi or ESj = Max (Esi + tij)

for all (i, j) activities, with ESi = O being the earliest start time of the beginning event of the project.

(b) Let us suppose that we have a target time for completing the project. Then this time is called the latest finish time (LF) for the final activity. The latest start time (LS) is the latest time at which an activity can start if the target is to be maintained.

Thus, if LFi is the latest finish time of all the activities emanting from event i and tij is the estimated time of the activity (i, j), then

LFi = Min (LFi – tij), for all

defined (i, j) activities.

2.2.5 Types of Floats

An activity is said to be critical if a delay in its start will cause a further delay in the completion of the entire project. On the other hand a non critical activity is such that the time between its ES and LF is longer than its actual duration. In this case, the non critical activity is said to have a slack or float time.

There are in general three types of floats, namely total float, free float and independent float.

(a) **Total float :-** This is calculated for any activity by using the following rules.

(i) Determine the difference between earliest start time of tail event and the latest finish time of head event for the activity.

(ii) Subtract the duration time of the activity from the value obtained in step (i) to get the required total float for the activity.

Thus the total float TFij for the activity (i, j) is obtained as

TFij = LFj – ESi - tij

(b) **Free float :-** It is defined by assuming that all the activities start as early as possible. The free float for the activity (i, j) is the excess available time over its duration. Thus

$FFij = ESj - ESi - tij$

(c) **Independent float :-** The time by which an activity can be rescheduled without affecting the preceding or the succeeding activities is known as independent float.

It is calculated as follows :

Independent float = Free float – tail event slack

2.2.6 CRITICAL PATH METHOD (CPM)

The iterative procedure of determining the critical path is as follows :

Step 1. List all the jobs and then draw a network diagram. Each job is indicated by an arrow with the direction of the arrow showing the sequence of jobs.

Step 2. Consider the jobs times to be deterministic.

Step 3. Calculate the earliest start time (EST) and earliest finish time (EFT) for each event. Also calculate the latest start time (LST) and latest finish time (LFT).

Step 4. Tabulate various times i.e. activity normal times, earliest times and latest times and mark EST and LFT on the arrow diagram.

Step 5. Determine the total float (slack) for each activity by taking the differences between EST and LFT.

Step 6. Identify the critical activities and connect them with the beginning node and ending node in the network diagram by double line arrows. This gives the critical path.

Step 7. Calculate the total project duration.

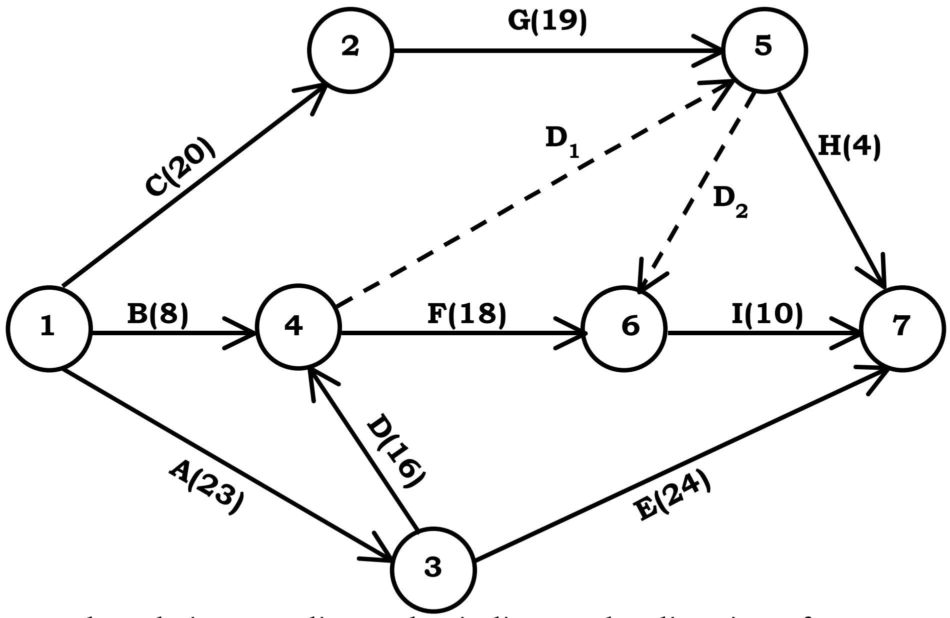
Example. A project consists of a series of tasks labelled A, B, ... H, I with the following relationships (W < X, Y means X and Y cannot start until W is completed ; X, Y < W means W can not start until both X and Y are completed). With this notation construct the network diagram having the following constraints :

$$A < D, E \; ; \; B, D < F \; ; \; C < G \; ; \; B < H \; ; \; F, G < I$$

Find also the minimum time of completion of the project, when the time (in days) of completion of each task is as follows :

Task	:	A	B	C	D	E	F	G	H	I
Time	:	23	8	20	16	24	18	19	4	10

Solution : Using the given constraints, the resulting network is shown below. The dummy activities are introduced to establish the correct precedence relationships. The events of the projects are numbered in such a way



that their ascending order indicates the direction of progress in the project :

------ Dummy activity

The critical path calculations are as follows :

$ES_1 = 0$

$ES_2 = ES_1 + t_{12} = 0 + 20 = 20$

$ES_3 = ES_1 + t_{13} = 0 + 23 = 23$

$$ES_4 = \underset{i=1,3}{\text{Max}} \left\{ Esi + ti4 \right\}$$

$= \text{Max } \{0 + 8, 23 + 16\} = 39$

$$ES_5 = \underset{i=2,5}{\text{Max}} \left\{ Esi + ti5 \right\}$$

$= \text{Max } \{30 + 0, 20 + 19\} = 39$

$$ES_6 = \underset{i=4,5}{\text{Max}} \left\{ Esi + ti6 \right\}$$

$= \text{Max } \{39 + 18, 39 + 0\} = 57$

$$ES_7 = \underset{i=3,5,6}{\text{Max}} \left\{ Esi + ti7 \right\}$$

$= \text{Max } \{23 + 24, 39 + 4, 57 + 10\} = 67$

The values of LFi are now obtained. These are :

$$LF_7 = ES_7 = 67$$

$$LF_6 = LF_7 - 6_{67} = 67 - 10 = 57$$

$$LF_5 = LF_5 = \underset{j=6,7}{\text{Min}} \begin{Bmatrix} LF_j - t5j \\ \end{Bmatrix} = \text{Min} \begin{Bmatrix} 57 - 0, 67 - 4 \\ \end{Bmatrix} = 57$$

$$LF_4 = \underset{j=5,6}{\text{Min}} \begin{Bmatrix} LF_j - t4j \\ \end{Bmatrix} = \text{Min} \begin{Bmatrix} 57 - 0, 57 - 18 \\ \end{Bmatrix} = 39$$

$$LF_3 = \underset{j=4,7}{\text{Min}} \begin{Bmatrix} LF_j - t3j \\ \end{Bmatrix} = \text{Min} \begin{Bmatrix} 39 - 16, 67 - 24 \\ \end{Bmatrix} = 23$$

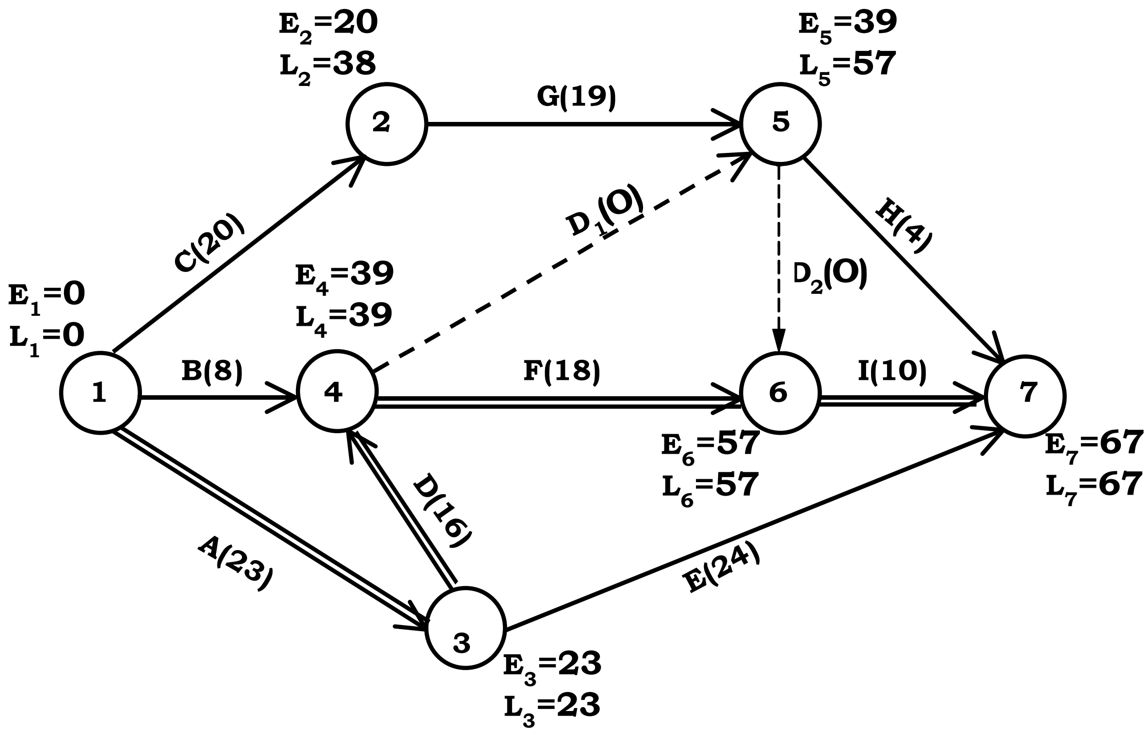
$$LF_2 = LF_5 - t_{25} = 57 - 19 = 38$$

$$LF_1 = \underset{j=2,3,4}{\text{Min}} \begin{Bmatrix} LF_j - t1j \\ \end{Bmatrix} = \text{Min} \begin{Bmatrix} 38 - 20, 23 - 23, 39 - 8 \\ \end{Bmatrix} = 0$$

To evaluate the critical events all these calculations are put in the following table.

Task	**Normal time**	**Earliest time**		**Latest time**		**Total**
		Start	**Finish**	**Start**	**Finish**	
(1,2)	20	0	20	18	38	18
(1,3)	23	0	23	0	23	0
(1,4)	08	0	08	31	39	31
(2,5)	19	20	39	38	57	18
(3,4)	16	23	39	23	39	0
(3,7)	24	23	47	43	67	20
(4,5)	0	39	39	57	57	18
(4,6)	18	39	57	39	57	0
(5,6)	0	39	39	57	57	18
(5,7)	04	39	43	63	67	24
(6,7)	10	57	67	57	67	0

The above table shows that the critical events are for the tasks (1, 3), (3,4),



(4,6) and (6,7)

It is observed from the figure that the critical path comprises the tasks A, D, F and I. This path represents the least possible time to complete the entire project.

2.2.7 PROGRAMME EVALUATION AND REVIEW TECHNIQUE (PERT)

The network method discussed so for may be termed as deterministic, since estimated activity times are assumed to be the expected values. Deterministic network methods assume that the expected time is the actual time taken. Probabilistic methods, on the other hand, assume the reverse, more realistic situation, where activity times are represented by a probability distribution. This probability distribution of activity time is based upon three different time estimates made for each activity. These are as follows :

to = the optimistic time is the shortest possible time to complete the activity if all goes well. That is, there is very little chance that activity can be done in time less than tp.

tp = the pessimistic time is the longest time that an activity could take place if every thing goes wrong. That is, there is very little chance that activity can be done in time less than t_p.

tm = the most likely time is the estimate of the normal time an activity would take. If only one were available, this would be it. Otherwise it is the mode of the probability distribution.

From these values it is necessary to derive the expected time. This is accomplished by an approximation developed by the experts of PERT and is given by

$$t_e = \frac{1}{3}\left\{2t_m + \frac{1}{2}\left(t_o + t_p\right)\right\}$$

$$= \frac{1}{6}\left\{4t_m + t_o + t_p\right\}$$

Another assumption concerning the distribution is that for unimodal distributions, the standard deviation can be estimated as one sixth of range, i.e.

$$\sigma = \frac{t_p - t_o}{6}$$

and therefore the variance is

$$\sigma^2 = \left(\frac{t_p - t_o}{6}\right)^2$$

2.2.8 PROBABILITY OF MEETING THE SCHEDULE TIME

With PERT, it is possible to determine the probability of completing a contract on schedule. The scheduled dates are expressed as a number of time units from the present time. Initially they may be the latest time, TL, for each event, but after a project is started we shall know that how far it has progressed at a given date and the scheduled times will be the latest times if the project is to be completed on its original schedule.

Since μi is the sum of independent variables, it is approximately normally distributed with mean $E(\mu i)$ and variance var (μi). Now, because the earliest expected time is μi, therefore the event i will meet a certain schedule time STi with probability :

$$P\left[\mu i \le STi\right] = P\left[\frac{\mu i - E(\mu i)}{\sqrt{Var(\mu i)}} \le \frac{STi - E(\mu i)}{\sqrt{Var(\mu i)}}\right]$$

$$= P\left[Z \le Z_0\right]$$

Where Z is the standard normal variate with mean zero and variance unity

and $$Z_0 = \frac{STi - E(\mu i)}{\sqrt{Var(\mu i)}}$$

Example : A small project is composed of seven activities whose time estimates are listed in the table as follows :

Activity		Estimated duration		
i	**j**	**Optimistic**	**Most likely**	**Pessimistic**
1	2	1	1	7
1	3	1	4	7
1	4	1	2	8
2	5	2	1	1
3	5	1	5	14
4	6	2	5	8
5	6	3	6	15

(a) Draw the project network.

(b) Find the expected duration and variance of each activity.

(c) Calculate early and late occurrence times for each event. What is the expected project length.

(d) Calculate the variance and standard deviation of project length. What is the probability that the project will be completed :

(i) at least 4 weeks earlier than expected.

(ii) no more than 4 weeks than expected.

(e) If the project due date is 19 weeks, what is the probability of meeting the due date ?

Given					
Z	0.50	0.67	1.0	1.33	2.0
P	0.3085	0.2514	0.1587	0.0918	0.0228

Solution : The expected time and variance of each activity is computed in table below.

Activity $(\mathbf{i}-\mathbf{j})$	t_0	t_m	t_p	$t_e = \dfrac{t_o + 4t_m + t_p}{6}$	$\sigma^2 = \left(\dfrac{t_p - t_0}{6}\right)^2$
1-2	1	1	7	2	1
1-3	1	4	7	4	1
1-4	2	2	8	3	1
2-5	1	1	1	1	0
3-5	2	5	14	6	4
4-6	2	5	8	5	1
5-6	3	6	15	7	4

Earliest expected times E (μi) for each event is obtained by taking the sum of expected times for all the activities leading to event i. When more than one activity leads to event i, the greatest of E(μi) is chosen. Thus, we have

$$E(\mu_1) = 0, E(\mu_2) = 0 + 2 = 2$$

$$E(\mu_3) = 0 + 4 = 4, E(\mu_4) = 0 + 3 = 3$$

$$E(\mu_5) = \text{Max}\{4 + 6, 2 + 6\} = 10$$

$$E(\mu_6) = \text{Max}\{10 + 7, 3 + 5\} = 17$$

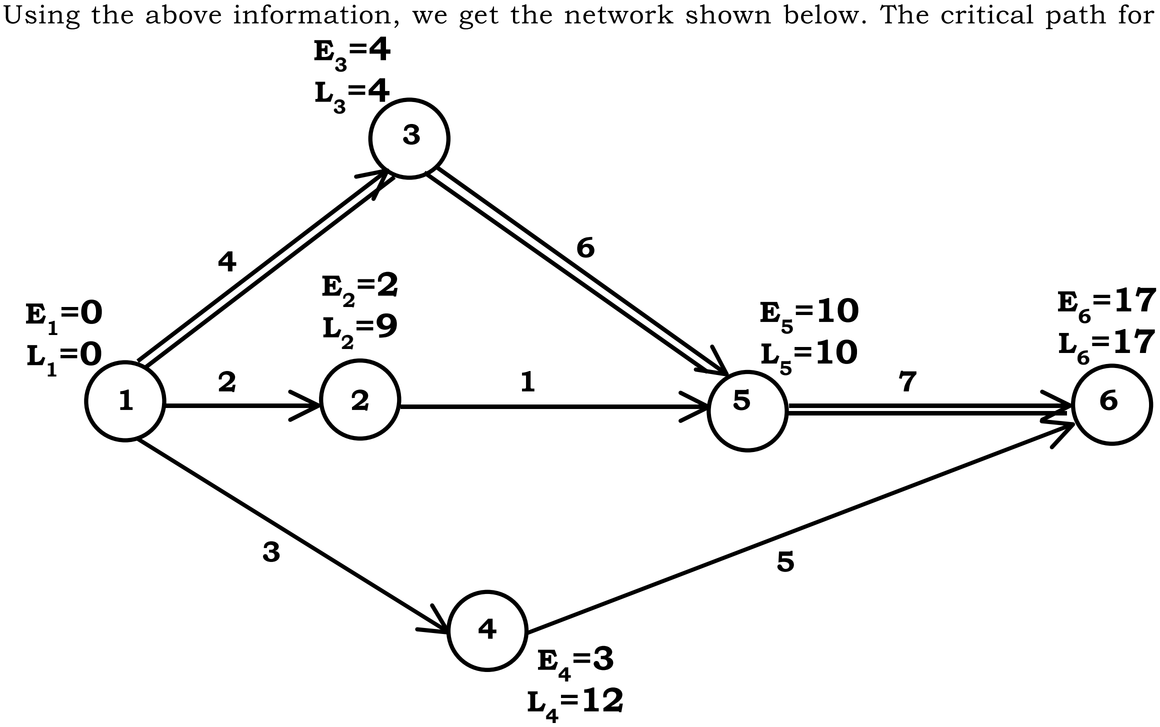
For the latest expected times we start with $E(\mu_6)$ for the last event and move backwards, subtracting 'te' for each activity link. Thus we have

$$E(L_6) = 17, E(L_5) = 17 - 7 = 10$$

$$E(t_4) = 17 - 5 = 12, E(L_3) = 10 - 6 = 4$$

$$E(L_2) = 10 - 1 = 9, E(L_1) = \text{Min}\{9 - 2, 4 - 4, 12 - 3\} = 0$$

Using the above information, we get the network shown below. The critical path for



the problem under consideration is shown by double line arrows.

Form the above diagram, we observe the following :

Critical path : $1 \rightarrow 3 \rightarrow 5 \rightarrow 6$

Other paths : $1 \rightarrow 2 \rightarrow 5 \rightarrow 6$ and $1 \rightarrow 4 \rightarrow 6$

The expected duration of the project is 17 weeks.

Variance of the project is given by

$$\sigma^2 = 1 + 4 + 4 = 9$$

The standard normal deviate is :

$$Z = \frac{\text{Due date} - \text{Expected date of completion}}{\sqrt{\text{Variance}}}$$

$\therefore$ We compute

(i) $$Z = \frac{12 - 17}{3} = \frac{-4}{3} = -1.33$$

$$P(Z \leq -1.33) = 0.5 - \phi(1.33)$$
$$= 0.5 - .4082$$
$$= 0.0918 = 9.18\%$$

The interpretation of the above is that if the project is performed 100 times under the same conditions, there will be 9 occasions when this job will be completed 4 weeks earlier than expected.

(ii) $$Z = \frac{21 - 17}{3} = \frac{4}{3} = -1.33$$

$\therefore$ The probability of meeting the due date (4 weeks later than expected) is

$$P(Z \leq 1.33) = 0.5 - \phi(1.33)$$
$$= 0.5 + .4082$$
$$= 0.9082 = 90.82\%$$

(e) When the due date is 19 weeks

$$Z = \frac{19 - 17}{3} = \frac{2}{3} = -0.67$$

$\therefore$ The probability of meeting the due date is

$$P(Z \leq .67) = .5 - \phi(.67) = .7486$$
$$= 74.86\%$$

Thus the probability of not meeting the due date is 1–0.7486 = 02514 = 25.14%

2.2.9 REFERENCES

1.	V.K. Kapoor	:	Operations Research techniques for management.
2.	Kanti Swarup, P.K. Gupta & Man Mohan	:	Operations Research

2.2.10 EXERCISE

1. Discuss the advantages and disadvantages of Network Techniques.
2. Write notes on :
   (a) Event and Activity
   (b) Phases of Project Management
3. A project schedule has the following characteristics

Activity:	1-2	1-4	1-7	2-3	3-6	4-5	4-8	5-6	6-9	7-8	8-9
Time :	2	2	1	4	1	5	8	4	3	3	5

   Construct the PERT Network and find the critical path and time duration of the project.
4. Three time estimates (in months) of all activities of a project are as given below :

Activity	t_0	t_m	t_p
1-2	3	4	5
2-3	6	8	10
2-4	2	3	4
3-4	4	5	12
4-5	5	7	9
5-6	9	16	17

   (a) Find the expected duration and standard deviation of each activity.
   (b) Construct the project network.
   (c) Determine the critical path, expected project length and variance of the project.

Type Setting :
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