Department of Open & Distance Learning

Punjabi University, Patiala

Class : B.Ed.-I **Semester : 2**

Paper : XI & XII (Teaching of Mathematics)

Medium : English **Unit: 1**

Lesson No.

1.1 Strategies for learning and teaching mathematics
1.2 Methods of teaching: Heurisrtic, Project, Laboratory
1.3 Method of Teaching-Problem Solving
1.4 Methods of teachng: Inductive-deductive, analytic-synthetic and the Van-Hiele levels of Geomatric thinking

Department website : www.pbidde.org

B.Ed. Part-I (Semester-II) | **Paper: XI & XII (ix)**
(Teaching of Mathematics)

Lesson No.-1.1 | **Author: Mrs. Reena Yadav**

Strategies for Learning and Teaching Mathematics

Structure of the Lesson

1.1.1 Objectives

After going through this lesson learners will be able to:-

i) define concept formation
ii) explain concept attainment
iii) explain fundamental of concept attainment model
iv) recall merits and limitations of concept attainment model
v) illustrate of concept attainment model in teaching of Mathematics
vi) explain constructivism and zone of proximal development
vii) differentiate between concept formation and concept formation

1.1.2 Introduction

Mathematics is considered a complex system of concepts. The learner should be given well devised and logical mathematical experiences and knowledge through teaching, so that they form new and exact concepts. To make mathematics clear, precise and enjoyable, concepts should be cleared from primary level to higher level. For clearing concepts, effective learning and teaching strategies should be used. Strategies mean a plan of action designed to achieve a long term or over all aim.

1.1.3 Concept Formation: Concept formation is an inductive teaching strategy that helps students forms a clear understanding of a concept (or idea) through studying a small set of examples of the concept. It is classification activity that leads the students to use item characteristics for classification. It develops their abilities to observe items thoroughly and to make useful observation. It also help them do discover methods of classification. Mathematics aims to develop critical thinking and reasoning in students.

Purpose: Concept formation as a teaching strategy is to have the student examine carefully some objects / actions / processes, and to think about a method for classifying them.

Aim : Concept formation helps to disregard what is inessential by creating idealized structures that focus on what is essential.

1.1.4 Concept Attainment Model

The term "Concept Attainment Model" is historically linked with the work of Jerone Bruner and his associates and that is why it is usually named as Bruner Concept Attainment Model.

1.1.4.1 Concept attainment: is an instructional strategy that uses a structured inquiry process. In concept attainment, students figure out the attributes of a group that has been provided by the teacher. To do so, students compare and contrast example that contain the attributes of the concept with examples that do not contain those attributes. By observing these examples, students discuss and identify the attributes of each until they develop a tentative hypothesis about the concept. Next, students separate the examples into two groups, those that have the attribute and those that don't. This hypothesis is then tested by applying it to other examples of the concept that could be symbols, words, passages, pictures or objects. This strategy can be used in all curriculum areas. Finally, students demonstrate that they have attained the concept by generating their own examples and non-examples. Concept attainment, then is the search for and identification of all attributes that can be used to distinguish examples of a given group or category from non examples.

1.1.4.2 Fundamental elements of concept attainment model:

Focus: The concept attainment model facilitates the type of learning referred to as conceptual learning, in contrast with the role learning of factual information or of vocabulary. In practice the model works as an inductive model designed to teach concept through the use of examples. Therefore, in addition to help the students in the attainment of a particular concept, the model also enables them to become aware of the process of conceptualizing.

Syntax: The sequence of the phases and activities covering the concept attainment Model may be outlined as below:

Table 1. Syntax of the concept attainment model

Phase One: Presentation of Data and Identification of the concept.

(Activities) (i) Teacher presents labelled examples (both positive and negative)

(ii) Students compare attributes of positive and negative examples

(iii) Students generate and test hypothesis.

(iv) Students name the concept, and state a particular definition of the concept

Phase Two: Testing Attainment of the concept.

(Activities) (i) Students identify additional unlabelled examples

(ii) Teachers name the concept

(iii) Students generate examples

(iv) Teacher restate definition, confirm hypothesis

Phase Three: Analysis of Thinking strategies.

(Activities) : (i) Students Describe thoughts

(ii) Students discuss role of hypothesis and attributes

(iii) Students discuss type and number of hypotheses

(iv) Teacher evaluate the strategies

Principles of Reaction: Important principles of reaction for the responses and regard of the students may be put as under:

1. The teacher has to remain supportive of the student’s hypotheses.
2. He/she has to maintain record by keeping track of the hypotheses.
3. He/she has is to remain supportive for turning the student’s attention toward analysis of their concepts and strategies.
4. He/she has is to encourage analysis of the merits of various strategies.

The Social System: In the concept attainment model the teacher is controller of the situation. The three major functions of the teacher are to record, promote and present additional data.

The system, as a whole, provides smooth interaction between teacher and the students for a cooperative sharing of the teaching and learning.

The Support System: Teaching with Concept Attainment Model requires some additional support in the following form:

1) Concept attainment lesson requires that both positive and negative examples be presented to the students.
2) As the students have to describe the attributes in the form of "yes" or "no", something like a black board or tagbord is required for recording these responses. The formulated hypothesis can be evaluated in the light of these attributes available in the class in a summarized form.

Application Context: Concept Attainment Model proves an excellent strategy to teach concept through the use of examples. It certainly helps the students to acquire concepts with which they are unfamiliar.

1.1.4.3 Merits of concept attainment model:

1 It is helpful in developing the power of imagination of the students.
2. It helps in the development of the reasoning power of the students.
3. It helps the students to analyze think systematically.
4. It helps the students to apply their knowledge in different situations.

1.1.4.4 Limitations: It makes high demands on the students as well as teachers. All the students of the class may not be able to participate in the teaching learning process. Some students, on account of their shyness, fail to derieve the requisite advantages of this model.

1.1.4.5 Illustration of a concept attainment model

1. First the teacher chooses a concept to developed. (i.e. Math facts that equal 20)
2. Begin by making list of both positive "yes" and negative " no" examples: The examples are put onto sheets of paper or flash cards.
3. Positive Examples: (Positive examples contain attributes of the concept to be taught) i.e. 10+10, 21-1, 20X1, 8+6+6, 22-2, 25-5, (8X2)+4, 19+1

4. Negative Examples: (for examples choose facts that do not have 20 as the answer) i.e. 7+6, 6+6, 22-4, 5X5, 6X4, 18-5, 7X2, 3+5+6, 2+(3X2), 26-10
5. Designate one area of the chalkboard for the positive examples and one area for negative examples. A chart could be set up at the front of the room with two columns - one marked YES and the other marked NO.
6. Present the first card by saying, "This is a YES." Place it under the appropriate column. i.e. 10+10 is a YES
7. Present the next card and say, "This is a NO." Place it under the NO column. i.e. 7+7 is a NO
8. Repeat this process until there are three examples under each column.
9. Ask the class to look at the three examples under the YES column and discuss how they are alike. (i.e. 10+10, 21-1, 2X10) Ask "What do they have in common?"
10. For the next three examples under each column, ask the students to decide if the examples go under YES or NO.
11. At this point, there are 6 examples under each column. Several students will have identified the concept but it is important that they not tell it out loud to the class. They can however **show** that they have caught on by giving an example of their own for each column. At this point, the examples are student-generated. Ask the class if anyone else has the concept in mind. Students who have not yet defined the concept are still busy trying to see the similarities of the YES examples. Place at least three more examples under each column that are student-generated.
12. Discuss the process with the class. Once most students have caught on, they can define the concept. Once they have pointed out that everything under the YES column has an answer of 20, then print a new heading at the top of the column (20 Facts). The print a new heading for the NO column (Not 20 Facts).

1.1.5 Constructivism

It is a learning theory found in psychology which explains how people might acquire knowledge and learn. It therefore has direct application to education.

Constructivism supports the notion that children learn effectively through interactions with experiences in their natural environment. Steffe and Killion stated that from a constructivist perspective, “Mathematics teaching consists primarily of the mathematical interactions between a teacher and children". This indirect approach to instruction effectively allows the student to learn in the context of meaningful activities. Learning is an endless, lifelong process that results from interactions with a multitude of situations. The constructivist approach does not solely focus on the action of the teacher or the learner, but on the interactions between the two. The teacher should make a conscious effort to see personal actions as well as the students from the student's point of view.

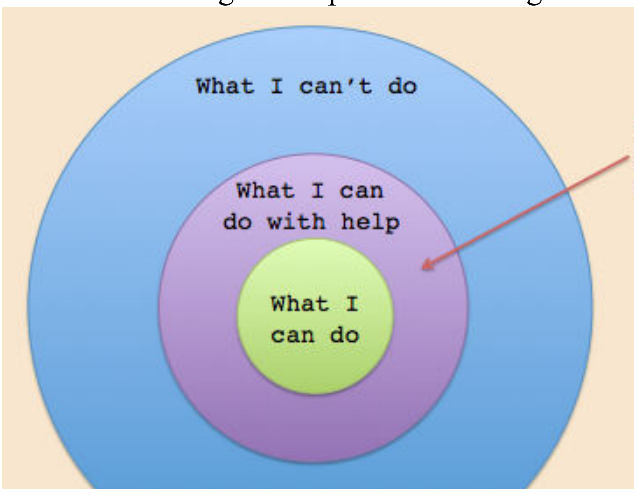
Constructivism has multiple roots in the psychology and philosophy of this century. There is no single concrete definition of constructivism. Piaget is thought to be one of the first learning theorists to advocate a constructivist teaching approach, even though he did not identify himself as such. He believed in the importance of human interaction and physical manipulation in the training of knowledge. The emphasis of the constructivist classroom begins with the student. The constructivist educator demonstrates a respect for the student. The classroom should be a place that fosters and nurtures learning and development of knowledge.

The constructivist classroom creates an environment that encourages learning. The teacher should create surroundings where students can make sense of mathematics as it relates to the real world. Students are to be treated with respect and responsibility. The fear of failing must be erased in order to foster the idea that students can learn from their mistakes. It we expect our students to comprehend and deal with complex problems, we need to establish an atmosphere rich with exposures. Students should become aware of their own thinking process, strategies, and critical thinking abilities. Each student should become aware of the ability to invent and explore new ideas and concepts. The effective teacher of mathematics must be willing to capitalize on these natural thinking abilities.

Constructivism focuses on student-centered instruction, which is not \new in education. The student plays a major role in the decision-making process as to what, when and how learning is to occur. This is a bold approach, but students have great insight into themselves that teachers cannot always see. We need to show respect to each student by providing the opportunity to shape individual learning.

1.1.6 Zone of Proximal Development

The zone of proximal development (ZPD), often referred to as the optimal learning zone, is a concept developed by social cognitive theorist and psychologist Lev Vygotsky. This is an important concept that relates to the difference between what a child can achieve independently and what a child can achieve with guidance and encouragement from a skilled or more knowledgeable partner. In figure shown below: ---.



Each student is unique in multiple and different zones depending on the content and an individual's prior knowledge and interests. Learning that occurs within the ZPD should neither be too easy nor too difficult. It needs to be just enough of a challenge to help the student deepen conceptual understanding and develop new skills that build upon the student's existing knowledge and understanding.

ZPD in the math classroom

Because the concept of ZPD involves connecting unlearned material to what is familiar, teachers play a pivotal role in the application of ZPD in the classroom. They provide appropriate scaffolding strategic social interactions, learning experiences, and instructions based on a student's past performance, intuition, and current thinking that guides effective learning and development.

Scaffolds facilitate a student's ability to make sense of new situations, build on prior knowledge, and transfer learning. There are many ways to scaffold to improve learning. In the math educational setting, scaffolds may include manipulative, games, models, cues, prompts, hints, partial solutions, think-aloud modeling, or using contextual problems based on a student's interests. Scaffolds should engage students in sense-making and critical thinking.

1.1.7 Summary: Many student love Mathematics in the starting but due to some reasons during the school years they kill that joy. With proper guidance, using different strategies with learning aids, a student can rediscover mathematical truth in reasonable time. They should made to think, instead being provided with readymade answers. They are encouraged to ask thought provoking questions. In this way, their reasoning, thinking and logical abilities should be developed. The teacher should take initiative of these activities. He should guide students to initiate of these activities. He can guide students to explore ideas, formulate concepts and generalizations, handle abstractions and the most important to make students thought process clear. Thus, he should play the role of an educator who not only makes children learn, but promotes creative thinking and scientific attitude in them.

1.1.8 Suggested questions

1. What is concept formation?
2. Explain concept attainment model in teaching of mathematics.
3. Differentiate between concept formation and concept attainment.
4. Explain constructivism and zone of proximal development for learning and teaching of concepts.

1.1.9 Suggested Reading:

1. Bruner, J., Goodnow, J. J., & Austin, G. A. (1967). A study of thinking. New York: Science Editions.
2. John Mason, Sue Johnston-Wilder (2004). Fundamental Constructs in Mathematics Education, Psychology Press, UK
3. K. S. Prabhakaram (1998). Concept Attainment Model In Mathematics Teaching, Discovery Publishing House, Delhi
4. Mujibul Hasan Siddiqui (2005). Teaching of Mathematics, APH Publishing Corp., New Delhi
5. Aggarwal, J.C. (1996). Essential of Education Technology: Teaching Learning, Vikas Publishing House Pvt. Ltd., New Delhi
6. Singh, Y.K. (2008) Educational Technology: Teaching and Learning, APH Publishing Corp., New Delhi
7. Mangal, S.K. (2001) Foundations Of Educational Technology, Tandon / Vinod Publications, Ludhiana (Punjab)

LESSON NO. 1.2 **Author: Dr. Kulbir Singh Sidhu**

Structure of the lesson:

1.2.1 Objectives

After going through this lesson learness will be able to-

- Know about the Heuristic method
- Explain merits and Drawback of Heuristic method
- Know about project method
- Explain the procedure of Project method
- Know about Laboratory method
- Explain merits and Draw backs of Laboratory method

1.2.2 Heuristic Method

The term 'heuristic' has been derived from the Latin word '*heurisco*' which means, 'I have found out'. It aims at transforming the learning process where in the students are not to be passive recipients but active and independent discoverers of knowledge. Staunch supporter of this method are of the view that every child should be made a discoverer and inventor. The teacher should stand aside as an onlooker and the learner should select his own path and proceed according to his own lights. The teacher's job is not to solve problems for the pupil, but to enable the pupil to solve problems for himself.

Prof. Armstrong originated this method. He was of the opinion that the school should develop thinking power of the student and for this the latter must be given an opportunity to think. This method is an attitude, now popularly known as scientific and heuristic attitude. Learner is trained in the art of learning. Self confidence, originality, independence of judgement and thinking power are to be developed in him to make an ever successful learner.

1.2.2.1 Procedure of the Method

Example : The diagonals of a parallelogram bisect each other. False heuristic type question will be :

(i) Is ABCD a parallelogram ?

(ii) Do the diagonal of a parallelogram bisect each other ?

Instead the true heuristic type question will be ?

(i) What type of ABCD ?

(ii) What do you know about the diagonal of such a figure ?

The students will be provided with many parallelograms already drawn on sheets of paper and the teacher will ask them to measure the diagonal and their parts. Their measurement and observation will enable them to generalize about the diagonals bisecting each other. They can be further encouraged to draw parallelograms and their diagonals and find the same characteristic by measurement.

1.2.2.2 Merits of Heuristic Method

1. The student becomes an active participant in the process of learner.
2. He thinks for himself and does not merely listen for information.
3. It is a psychologically sound method as it uses the active original and constructive tendencies of the learner.
4. After discovering something by self effort, the students take pride in his achievement and this feeling encourages him towards more achievement.
5. The teacher remains in constant touch with his students.
6. It develops in the student, scientific attitude or heuristic attitude.
7. The students acquires clarity about the subject. He gets complete mastery over what he has learnt.
8. As the responsibility of learning is entrusted to the students, this method is helpful in student discipline.

Drawbacks of Heuristic Method

1. It demands special preparation and greater labour from the already over burdened teacher.

2. Every teacher may not be equally competent to use this method. Some of the teachers may not be gifted with heuristic spirit.
3. It is a slow method. Investigation is likely to take much time.
4. The learner let to himself may not be bale to make study and adequate progress.
5. In lower cases, especially, the child needs guidance and hints. If the teacher denies him proper guidance, he may get discouraged.
6. Every child may not be a gifted discoverer. The immature has his own difficulties and limitations.
7. The teacher has to provide well measured guidance. Guidance in excess may harm the learner's initiative. Inadequate guidance may also harm his progress.
8. If the students are temped to consult books and copy for the sake of discovery, the method fails in its purpose.
9. This method may not be applicable to all the topics equally well.
10. The method pre-supposes small classes as the teacher is required to give individual attention to the students.
11. The success of this method depend largely on the teacher's art of questioning; bad questions cannot provoke real thinking.
12. The teacher may also fail to distinguish between false heuristic and true heuristic; questions in that case the method degenerates into worthless questioning.
13. There is the possibility of erroneous conclusion's on the part of the students.
14. It needs a lot of hard work, patience, concentration, sound thinking and creative abilities. Every child may not possess these qualities.
15. There is no literature written on heuristic lines. Adequate library and laboratory facilities may also be lacking.

Conclusion

The extreme form of this method is out of question. The teacher will have a meaningful presence in the classroom. He will not be an indifferent onlooker. His presence must inspire and stimulate the learner. In practice, this method will be based on good questioning. Nothing is to be told so long as the student goes on answering questions. As far as possible, he has to be allowed to progress by himself. His difficulties and limitations should not be allowed to stand in his way. The teacher has to frame questions, instructions and hints very carefully so as to avoid overfeeding. He should let the child be his own teacher and also see that his difficulties are removed in time. Whatever be his method of teaching, the guiding principle should be the adoption of heuristic approach.

1.2.3 Project Method

Project method is the outcome of John Dewey's philosophy of pragmatism. He says "What is to be taught should have a direct relationship with the actual happenings in life," In order to understand this method, let us try to think over the meaning of the term "project".

According to Stevenson–"A project is a problematic act carried its completion in its most natural setting".

According to Kilpatrick–"A project is a whole-hearted purposeful activity proceeding in a social environment".

According to Ballard—"A project is a bit of real life that has been imported in to school". These definitions present to us the following characteristics of a project :

1. A project is an act related to actual life activities.
2. This act is undertaken to solve an emerging or felt problem in order to realize some useful and purposeful objectives.
3. It is always accomplished in a social environment and a natural setting.
4. The project activity will remain interesting and absorbing throughout for the learner.

Project plan is a more modified form of an old method 'concentration of studies'. The main feature of this plan is that some subject is taken as the core or center and all their school subjects, as they arise, are studied in connection with it. It is also based on the principle of learning by doing. It assumes that knowledge grows by application. It is also based on the fact that the different branches of knowledge are not separable, though they are studied separately for some superficial convenienced. Knowledge is invisible, and project method is a method in accordance with this natural correlation. It is method of spontaneous and incidental teaching. As the project progresses, the learner go on picking up any piece of knowledge that may happen to be relevant, necessary and useful.

1.2.3.1 Procedure :

The carrying out of a project involves the following six steps :

1. Providing a situation.
2. Choosing and purposing.
3. Planning of the project.
4. Executing the project.

5. Evaluation of the project.
6. Recording of the project.

1. Providing a Situation :

In the first step, a situation is provided to the students to think over in choosing some project to work on.

2. Choosing and Purposing :

In this step students try to choose a definite project keeping in view of the resources in hand. They are properly guided by their teachers in the selection of a project.

3. Planning of the Project :

In the planning phase, the project chosen is again discussed in items of laying down a plan and procedure for the execution of the project.

4. Executing the Project :

At this stage the project is in process. All the students play their roles according to their abilities and capacities.

5. Evaluation of the Project :

The work done on the project is evaluated from time to time.

6. Recording of the Project :

In this step the students note down the whole process of the project, i.e. how the project was chosen, planned and executed, what type of difficulties were faced and how they were solved. This is kept as record for the future guidance.

There cannot be any rigidity about these steps and stages. Modification and adjustments can be made according to the nature of the project and the level of the students. The teacher has to make among the pupils an equitable distribution of work, according to their abilities and stamina. On the whole the project work remains a cooperative activity of the teacher and the taught and it is joint venture before the students.

A project should arise out of a need felt by pupils. It should not be forced on them. It should be important and purposeful. It must be interesting for the students.

Ex. 'Understanding about a local industry' may be a project for the students. The following aspects may have to be dealt with for the accomplishment of the project.

1. The name of the industry (e.g., sports goods)
2. Location of various important factories engaged in the industry, and the names of the concerns.
3. The reasons justify the location of the industry in that area.

4. The source of its raw material.
5. The geography of its raw material.
6. The expansion of the industry over a span of time.
7. Whether factory is individual enterprise, join concerns or cooperative societies.
8. Approximate number of people employed in that industry.
9. Their ranks, grades and salaries.
10. Annual turn out of various factories.
11. The share of profit for different share holders.
12. The progress of different factories in the matter of production and income from year to year. The graphs of this progress.
13. The scope of further expansion of the industry.
14. The relation of the industry with other social institutions.
15. The contribution of the industry in the overall development of the area.
16. The type of qualifications–general and technical which can enable a person to get employment in that industry.
17. Any scientific, chemical or mechanical processes involved in various factories.
18. Visit to important factories.
19. An essay on this local industry.
20. A quiz competition on the local industry.

While answering these and many other possible queries, the students will acquire a through knowledge of local industry and at the same time learn many relevant topics of different subjects. The teaching of relevant mathematics will also come in incidentally.

1.2.3.2 Merit of Project Method

1. Better learning takes place in a project plan because the pupils get the opportunity of active participation and creative self expression.
2. The project enables the pupil to apply the principles learnt in school of life situations.
3. It is interesting to the pupils because it is an active programme.
4. It is based on the psychological laws of learning.
5. Education is related to life and is acquired through meaningful activity.
6. It upholds the dignity of labour.
7. There is an ample scope for the transfer of training.

8. It introduces democracy in education because the studies have to cooperate and act together for a common cause.
9. It brings about concentration of studies and correlation between knowledge and activity.
10. It emphasizes problem solving rather than cramming or memorizing.
11. It inculcates social discipline through joint activities.
12. It develops self-confidence and self-discipline.
13. Teaching becomes incidental as the child is motivated by the desire to learn.
14. A project tends to illustrate the real nature of a subject and its application.
15. Project can be used to arouse interest, justify the study, encourage initiative and give the students joy at the successful completion of a given work.
16. It poses a challenge to the capacities and abilities of the learner and puts him on the track to think and act.
17. It provides opportunities of mutual exchange of ideas.
18. It develops a number of social virtues like sense of responsibility, resourcefulness self-respect, tolerance, patience etc.
19. Even backward children learn things with ease.

1.2.3.3 Drawbacks of Project Method

1. Project method creates many challenges for the teacher right from the selection of the project till its execution. Every teacher is not adequately equipped and informed to provide required enthusiasm, and leadership for the carrying out of a project.
2. Mathematics cannot be taught adequately by this method. Incidental teaching does not suffice in this subject which needs well-organized, systematic and continuous teaching. There are so many branches, topics and aspects of Mathematics that may hardly be covered through projects.
3. The drill and practice work which is the backbone of the Mathematics teaching cannot be provided through project method.
4. Through long drawn out project, hardly a part of the syllabus can be covered, therefore it does not suitably replace the present day speedily classroom teaching.
5. Our schools can neither afford sufficient funds nor provide

competent personnel for teaching through project method.

6. Suitable books are not available which may help the teacher to follow this method.
7. A limited opportunity available in a project for practical experience cannot develop skill, speed and efficiency in problem solving.
8. Our educational structure is examination oriented. The project method does not prepare the students adequately for the examination.

Conclusion

This method brings the life to the school atmosphere. Learning becomes a pleasure and a cooperative activity. Its approach is quite scientific and psychological. Irrespect of having so many points in its favour, the project method suffers from so many handicaps and limitations, as it is not suitable for drill, problem solving efficiency and systematic teaching. It is not very desirable to use it freely. The present classroom teaching can not be replaced by project work. However, if the teacher can devise and plan a good project on something, the students will gain a lot. Mathematics must be frequently taught in the way it is utilized in our practical life. Here is lies the need of working out some useful and productive activity in the form of projects. A wise teacher should employ project for the teaching of real and useful Mathematics. Project will be useful to show the application of new tool, application of a new formula, application of the subject life and produce spirit of enquiry and self reliance.

Examples

A few projects suitable for the subject of Mathematics are listed here for the teacher's guidance :

1. Running a school bank.
2. Running a cooperative store.
3. Collecting data about municipal, provincial and national budgets.
4. Graphs of employment, population etc. in the locality.
5. Mathematics in the school campus.
6. Use of Mathematics in large business.
7. Bus and railway fares from their locality to important stations.
8. Running the hostel mess.
9. Purchase of items for the school.
10. Collecting rates of commodities from a number of sources in the bazaar.
11. Model of the town.

12. Model of the state.
13. A picnic, its organization, expenditure, etc.
14. The sports day.
15. Planning and constructing a house.
16. Speed records of cycle, scooter, car, bus, train and aeroplane over a number of years.
17. The uses of Mathematics in a large establishment.

1.2.4 Laboratory Method

In this method we try to make the students learn Mathematics by doing experiments and laboratory work in the mathematics laboratory or room. It is on the same lines as the students learn Science by performing experiments.

This method involves the maxims of teaching like 'learning by doing', 'concrete to abstract', 'learning by observation' etc. It provides a practical base to inductive reasoning. The subject is learnt by doing rather than by reading. The doing of Mathematics necessitates as a suitable method and a suitable place. The activity involved in this method leads the pupil to discover mathematical facts. Principles are discovered, generalized and established. This method should help in the removal of the abstract nature of Mathematics. It makes the subject interesting as it combines play and activity.

J.W.A. Young has said, "The laboratory method aims to arouse teachers to a belief, not only theoretical but practical and effective as well that mathematical dishes must be made appetizing and palatable. They are to be accepted with pleasure and digested with ease." He has further remarked, that "a room specially filled with drawing instruments, suitable tables and desks, good black-boards, and the apparatus necessary to perform the experiments of the course are really essential for the best success of the laboratory method.

1.2.4.1 Procedure

Example : For calculating the area of a triangle, cut out a triangle of cardboard. Find the weight of a unit area of the cardboard and then find the weight of the triangle. The weight of the triangle divided by the weight of the unit area of the cardboard of same thickness will give you the area of the triangle.

Example : Calculation of the volume of a solid, like cuboids or cylinder can be done with the help of a graduated cylinder filled with water to a certain height. When we immerse the solid in this water, the rise in water will give the volume of the solid. After finding practically the volume of number of solids of one type, the formula can be established.

1.2.4.2 Merits of the Method :

1. Learning by doing is joyful for the learner.
2. It is a psychological method as it is based on maxims like 'learning by doing', 'learning from concrete to abstract'.
3. By discovery through self-effort, the learner acquires a clear understanding of the subject.
4. It provided good opportunity for independent work and individual development.
5. A successful experiment is a source of inspiration and encouragement to the learner.
6. It inculcates the spirit of co-operation and exchange of experience.
7. Shyness of hands is removed due to the handling of material and apparatus.
8. The learner observes application of Mathematics and thus the subject becomes functional and meaningful to him.
9. Some of the topics of Mathematics can be taught best by this method.
10. It ensures intimate contracts between the teacher and the taught.
11. It is a scientific method.

1.2.4.3 Drawbacks of the Method

1. The method will prove to be expensive. An average school cannot afford to spend large amounts on mathematics laboratory.
2. Laboratory method does not suit the subject of Mathematics. It does not provide for typical problem–solving, and does not give any training, in mathematical thinking.
3. It has only partial usability in the teaching of Mathematics. It can be used only for the topics where experimentation is possible.
4. It does not provide for mathematical reasoning and mental development.
5. It places too much expectation on the pupils. Immature students can not be expected to investigate or discover things independently like scientists.
6. Method demands thorough planning and supervision from the side of the teachers, otherwise students may just play with instruments without deriving any substantial gain.
7. The teacher will be required to pay individual attention which may not be possible in large classes.
8. It will prove to be a laborious and slow method.

9. Suitable text books written on the lines of laboratory method are not available. It is a hurdle for the teachers and students both.
10. Especially in lower classes the students may not be able to discover mathematical fact experimentally.
11. In higher classes also abstract concepts have to be developed. Therefore, it is not profitable to learn or to teach everything in a concrete form.
12. The method may degenerate into pure manual training only.
13. The tendency of cooking up result on copying may develop among the students.

Conclusion :

The construction work in Geometry is on the whole a laboratory work. Every proposition can also be proved practically before giving its theoretical proof, although it is a difficult and time consuming method. It can prove useful only if properly employed. Its success largely depends on facilities available in the form of staff, accommodation and equipment. Young children will be especially fascinated by this method. Its use should be must where circumstances favour.

1.2.5 SUGGESTED QUESTIONS

1. Discuss the heuristic method of teaching Mathematics. Illustrate its procedure by giving three examples.
2. What is the place of laboratory method in the teaching of Mathematics? What precautions do you suggest in its actual practice ?
3. How will you employ project method for the teaching of mathematics ? Illustrate with the help of suitable examples.
4. Discuss heuristic method of teaching with reference to its procedure, merits, drawbacks and applications.
5. What do you understand by laboratory method of teaching ? What are the practical difficulties in introducing laboratory work in mathematics in our school ?
6. How far can laboratory method be successfully employed in teaching Geometry ?
7. What are the strengths and limitations of the project method as far as teaching of mathematics is concerned ? Mention a suitable project and give details of carrying it out.
8. Write short notes on :
   (i) Limitations of heuristic method.
   (ii) Procedure of project method.

(iii) Merits of laboratory method.

1.2.6 REFERENCE BOOKS

1. *The Teaching of Mathematics* by K.S. Sidhu, Chapter Seven, Pages 120-137.
2. *A Course in Teaching of Mathematics* by S.M. Aggarwal, Chapter Six, Pages 78-103, 3, A.
3. *Text Book on Teaching of Mathematics* by S.K. Mangal, Chapter Seven, Pages 52-80.
4. *Teaching of Modern Mathematics* by S. Packiam, Chapters Four and Five, Pages 14-30.

1.	Schultze, A	:	*The Teaching of Mathematics in Secondary Schools.*
2.	Potter, F.F.	:	*The Teaching of Arithmetic.*
3.	Spitzar	:	*The Teaching of Arithmetic.*
4.	Aiyangar, N.K.	:	*The Teaching of Mathematics in the New Education.*
5.	Ballard, P.B.	:	*Teaching of Essentials of Arithmetic.*
6.	Bryan, Thwartes	:	*On Teaching Mathematics.*
7.	Butler and Wren	:	*The Teaching of Secondary Mathematics.*
8.	Davis R. Davis	:	*The Teaching of Mathematics.*
9.	Sexana, R.C.	:	*Curriculum and Teaching of Mathematics in Secondary Schools.*
10.	Young, J.W.A.	:	*Teaching of Mathematics.*

Lesson . No. 1.3 **Author: Mrs. Ishtdeep Kaur**

1.3.1 After reading this lesson students will be able to:

1. Describe the demonstration method in detail.
2. Explain the advantages and disadvantages of demonstration method in mathematics.
3. Discuss the contributions of problem solving method in the life of students.

1.3.2 What is Problem Solving?

Naturally enough, Problem Solving is about solving problems. And we'll restrict ourselves to thinking about mathematical problems here even though Problem Solving in school has a wider goal. When you think about it, the whole aim of education is to equip children to solve problems. In the Mathematics Curriculum therefore, Problem Solving contributes to the generic skill of problem solving .

On the other hand, the processes of mathematics are the ways of using the skills creatively in new situations. Problem Solving is a mathematical process. As such it is to be found in the Strand of Mathematical Processes along with Logic and Reasoning, and Communication. This is the side of mathematics that enables us to use the skills in a wide variety of situations.

Before we get too far into the discussion of Problem Solving, it is worth pointing out that we find it useful to distinguish between the three words "method", "answer" and "solution". By "method" we mean the means used to get an answer. This will generally involve one or more Problem Solving Strategies. On the other hand, we use "answer" to mean a number, quantity or some other entity that the problem is asking for. Finally, a "solution" is the whole process of solving a problem, including the method of obtaining an answer and the answer itself.

method + answer = solution

Polya's Four-Step Process

Probably the most famous approach to problem solving is Polya's four-step process described below (Polya, 1945). Polya identifies the four principles as follows:

1. Understand the problem
2. Devise a plan
3. Carry out the plan
4. Look back

Step 1: Understand the problem

To correctly solve a problem, you must first understand the problem. Below are some questions that may help lead you (or a student) to an understanding of a given problem. Not every question will be appropriate for every problem.
What are you asked to find or show?
What type of answer do you expect?
What units will be used in the answer?
Can you give an estimate?
What information is given? Do you understand all the terms and conditions?
Are there any assumptions that need to be made or special conditions to be met?
Is there enough information given? If not, what information is needed?
Is there any extra information given? If so, what information is not needed?
Can you restate the problem in your own words?
Can you act out the problem?
Can you draw a picture, a diagram, or an illustration?
Can you calculate specific numerical instances that illustrate the problem?

Step 2: Devise a plan

Use problem-solving strategies. The "plan" used to solve a problem is often called a problem-solving strategy. For some problems, you may begin using one strategy and then realize that the strategy does not fit the given information or is not leading toward the desired solution; in this case, you must choose another strategy. In other cases you may need to use a combination of strategies. Several problem-solving strategies are described below (in no particular order). All of these strategies can be found in elementary-level mathematics textbooks at around the third- or fourth-grade level; many of these strategies are introduced as early as the prekindergarten level.

- **Use guess and check.** When a problem calls for a numerical answer, a student may make a random guess and then check the guess with the facts and information given within the problem. If the guess is incorrect, the student may make and check a new guess. Each subsequent guess should provide more insight into the problem and lead to a more appropriate guess. In some instances the guess and check strategy may also be used with problems for which the answer is non-numerical.

- **Draw a picture or a diagram/use a graph or number line.** A picture or graph may illustrate relationships between given facts and information that are not as easily seen in word or numerical form.
- **Use manipulatives or a model/act it out.** When a problem requires that elements be moved or rearranged, a physical model can be used to illustrate the solution.
- **Make a list or table.** A list or table may be helpful to organize the given information. It may be possible to make an orderly list or table of all possible solutions and then to choose the solution that best fits the given facts and information from this list. In some problems, the answer to the problem is a list or table of all possible solutions.
- **Eliminate possibilities.** When there is more than one possible solution to a problem, each possibility must be examined. Potential solutions that do not work are discarded from the list of possible solutions until an appropriate answer is determined.
- **Use cases.** It is possible to divide some problems into cases. Each case may be separately considered.
- **Solve an equivalent problem.** In some instances it is easier to solve a related or equivalent problem than it is to solve a given problem.
- **Solve a simpler problem**. It may be possible to formulate and solve a simpler problem than the given problem. The process used in the solution of the simpler problem can give insight into the more complex given problem.
- **Look for a pattern.** Patterns are useful in many problem-solving situations. This strategy will be especially useful in solving many real-world problems. "Patterns are a way for young students to recognize order and to organize their world"
- **Choose the operation/write a formula or number sentence.** Some problems are easily solved with the application of a known formula or number sentence. The difficulty often lies in choosing the appropriate formula or operation.
- **Make a prediction/use estimation.** One must closely consider all elements of a problem in order to make a prediction or use estimation. This careful consideration may provide useful insight into the problem solution.
- **Work the problem backward.** If the problem involves a sequence of steps that can be reversed, it may be useful to work the problem backward. Children at the early childhood level may already have some experience in working backward. In solving many mazes and puzzles, it is sometimes easier to begin at the end than to begin at the beginning.
- **Use logical reasoning.** Mathematics can and should make sense. Logical reasoning and careful consideration are sometimes all that is required to solve a mathematics problem.

Step 3: Carry out the plan

Once a problem has been carefully analyzed and a plan is devised, if the plan is a suitable one for the given problem, it is usually a relatively simple process to carry out the plan. However, in some cases the original plan does not succeed and another plan must be devised. The original strategy may need to be modified, or a new strategy may be selected. Students must realize that not every problem will be solved within the first attempt. A failed attempt can be viewed as a learning experience. Try to help students avoid getting frustrated or discouraged. Cooperative learning teams can be used to encourage and engage students. Computers, calculators, or other manipulatives may be useful tools when routine tasks are involved.

Step 4: Look back

Once an answer or solution is found, it is important to check that solution. Check all steps and calculations within the solution process. Below are some questions that you (or your students) may find useful in the looking back process.
Is the answer reasonable?
Is there another method of solution that will easily verify the answer?
Does the answer fit the problem data?
Does the answer fulfill all conditions or requirements of the problem?
Is there more than one answer?
Will the solution process used be valuable in solving similar or related problems?

Step 5: Extend the problem

For a classroom teacher, an important part of the problem-solving process should involve trying to create similar or related problems. A given problem may need to be simplified in order to be used at a specific classroom level or with students that have special needs. A teacher may wish to make a problem more complicated or to create similar related problems that are more difficult. Elementary school students often extend the problem as part of a journal writing exercise as they write their own story problems for a given situation.
It may be possible to generalize specific instances of a given problem. Teachers must be on the lookout for opportunities to have students generalize and make conjectures. Teachers should look for connections that can be made between given mathematics problems and solutions and real-life situations. Teachers should also look for connections between given mathematics problems and their solutions and other subject areas. There are many excellent articles that deal specifically with posing problems and extending textbook exercises.

One of the aims of teaching through problem solving is to encourage students to refine and build onto their own processes over a period of time as their experiences allow them to discard some ideas and become aware of further possibilities (Carpenter, 1989). As well as developing knowledge, the students are also developing an understanding of when it is appropriate to use particular strategies. Through using this approach the emphasis is on making the students more responsible for their own learning rather than letting them feel that the algorithms they use are the inventions of some external and unknown 'expert'. There is considerable importance placed on exploratory activities, observation and discovery, and trial and error. Students need to develop their own theories, test them, test the theories of others, discard them if they are not consistent, and try something else (NCTM, 1989). Students can become even more involved in problem solving by formulating and solving their own problems, or by rewriting problems in their own words in order to facilitate understanding. It is of particular importance to note that they are encouraged to discuss the processes which they are undertaking, in order to improve understanding, gain new insights into the problem and communicate their ideas (Thompson, 1985, Stacey and Groves, 1985).

Conclusion

It has been suggested that there are many reasons why a problem-solving approach can contribute significantly to the outcomes of a mathematics education. Not only is it a vehicle for developing logical thinking, it can provide students with a context for learning mathematical knowledge, it can enhance transfer of skills to unfamiliar situations and it is an aesthetic form in itself. A problem-solving approach can provide a vehicle for students to construct their own ideas about mathematics and to take responsibility for their own learning. There is little doubt that the mathematics program can be enhanced by the establishment of an environment in which students are exposed to teaching via problem solving, as opposed to more traditional models of teaching about problem solving. The challenge for teachers, at all levels, is to develop the process of mathematical thinking alongside the knowledge and to seek opportunities to present even routine mathematics tasks in problem-solving contexts.

Advantages and disadvantages of using a group to solve a problem:

The disadvantages of group problem solving can include:-

Competition

Most people working in a group unconsciously perceive the situation as competitive. This generates behaviour which is destructive and drains the creative energy of the group. For example, we often perceive disagreement with our ideas as a put-down. The natural reaction is to regain our self-esteem, often by trying to sabotage the ideas of those who disagreed with us. Instead of looking for ways to improve on their ideas we choose to destroy them.

Eager to express our own ideas, we may totally ignore what others are suggesting. Power-seekers may use ploys such as highlighting flaws in others' arguments, barbed questions and displays of expertise to show their supremacy. These types of behaviour create an atmosphere which is incompatible with effective problem solving.

Conformity

There is a strong tendency for individuals in a group to want to conform to the consensus. This can be for a variety of reasons, including the need to feel liked, valued or respected, and tends to make people censor their ideas accordingly. The comparative status of the individuals present also has an important influence. Senior members often want to maintain their image of being knowledgeable, while junior members want to avoid appearing the inexperienced 'upstart'. Because agreement on ideas can be gained quickly in a group setting, groups tend to select and approve solutions quickly, without exploring all the possibilities.

Lack* of *objective direction

Most traditional meetings and group discussions convened to solve problems are ineffectively directed. Sometimes there is no effective leader to give direction to the discussion, with the result that it wanders aimlessly. Even when there is strong leadership, the group leader or chairman often exerts undue pressure on the direction and content of the discussion. In addition, the ideas aired during a meeting are not usually recorded, apart from the minutes and individual note-taking, with the result that many ideas are forgotten and cannot act as a constant stimulus to the discussion.

Time constraints

Group problem solving is a relatively slow process compared with working alone. It requires individuals to come together at an agreed time, usually for about one hour, and this can cause organisational problems as well as impatience amongst participants to 'get it over with' as quickly as possible.

The advantages of group problem solving can include:

Greater output

Simply because of the number of people involved, each with differing experience, knowledge, points of view and values, a larger number and variety of ideas for solving a problem can be produced.

Cross *fertilisation*

The exchange of ideas can act as a stimulus to the imagination, encouraging individuals to explore ideas they would not otherwise consider.

Reduced bias

The shared responsibility of a group in arriving at decisions can. encourage individuals to explore seemingly unrealistic ideas and to challenge accepted ways of doing things. Individual biases and prejudices can be challenged by the ,group, forcing the individual to recognise them. Group pressure can also encourage individuals to accept that change is needed.

Increased risk taking

Shared responsibility makes individuals more willing to take risks. The discussion of different points of view also helps the group to be more realistic in assessing the risks associated with particular courses of action.

Higher commitment

When goals are agreed it gives a common purpose to the group, within which individuals can gain a feeling of self-determination and recognition through their contribution. Individuals who have contributed to finding a solution feel a greater commitment to its successful implementation.

Improved communication

When .people who are affected by a problem or who will be involved in implementation are involved in finding a solution, they will know how and why that particular solution was chosen. Also, people with knowledge relevant to the problem can communicate that knowledge directly if they participate in solving the problem.

Better solutions

Groups of individuals can bring a broad range of ideas, knowledge and skills to bear on a problem. This creates a stimulating interaction of diverse ideas which results in a wider range and better quality of solutions.

3.4 Suggested Questions:-

1. Define Problem Solving Method.
2. Discuss the merits of Problem Solving Method.
3. Explain demonstration method.

1.3.5 Suggested books and web sources:-

1. www.studylecturenotes.com/...../demonstration-method-of-teaching
2. https://en.wikipedia.org/wiki/Demonstration_(teaching)
3. www.authorstream.com/..../november20-925733-approaches-inteaching
4. https://upwp2008.wikispaces.com/file/view/Teaching+Demontration.com
5. nzmaths.co.nz/what-problem-solving
6. www.education.com>School and Academics>Math Homework Help
7. www.mathgoodies.com/articles/problem_solving.htmls

LESSON NO. 1.4 Author: Dr. Kulbir Singh Sidhu

Sturcture of the lesson

1.4.1 Objectives

After going through this lesson learners will be able to-

- Know about Inductive-Deductive method
- Differentiate between inductive-Deductives method
- Know about analytic-synthetic method
- differentate between analytic -synthetic method
- Recall the term The Van - Hiele Levels of Geometric thinking
- Know about the kinds of proof
- Explain the direct proof
- Explain the proof by mathematical induction
- Explain proof by contradiction
- Explain disproof by counter example

1.4.2 Introduction

There are a quite number of methods available which can be profitably utilized in the teaching of Mathematics. None of the methods can be used exclusively. A successful teacher will try to be a master of all of them so that he can use any one of them with equal effectiveness when the situation demands. These methods are being discussed in the lesson third and fourth one by one in order to enable the student-teacher to employ them in proper situation's in his day to day teaching.

4..3 Inductive-Deductive Methods

These two methods largely work in combination in the teaching of Mathematics. We can understand their combined functioning better after discussing them separately.

Inductive method

In this method we proceed from particular to general, examples to general rule and from concrete to abstract. The mathematical formula is constructed in this method after understanding or solving a sufficient number of concrete examples. The universal truth of the formula is proved by introduction after showing that if it is true of a particular case. It is true of all such cases. The generalization is arrived at through a process of reasoning and solving of problems. After understanding a number of concrete cases, the student can successfully attempt generalization. When a general statement is made from a series of a particular cases, knowledge is born.

Procedure :

Ex.1 Ask the students to draw different parts of interesting straight lines. let them measure vertically opposite angles in each case. They will find them equal in all cases. The same results in a large number of cases will enable them to formulate the relevant generalization about the equality of vertically opposite angles.

Ex.2 Let the students find squares in the cases like a-b, x-y and p-q by the method of multiplication. They can be guided to generalize on the basis of these results that $(1\text{st term}-2\text{nd term})^2 = (1\text{st term})^2 + (2\text{nd term})^2 - 2\ (1\text{st term}) \times (2\text{nd term})$.

Merits of The Inductive Method

1. The concrete example are clearly perceived and make a deep and lasting impression on the mind of the learners.
2. Various examples enable pupils of different levels of understanding to participate actively in the process of learning.
3. It is a psychological method.
4. The examples show where the knowledge gained may be applied. It prepared the pupils to use it effectively whenever a similar situation presents itself.
5. Examples are more interesting to the learner. This interest enables the learner to give better attention to his work.
6. It suits the subject of mathematics as it is a logical method of arriving at rules and formulae.
7. It checks the tendency to learn things by role and may reduce repetition and home work.

Drawbacks of The Inductive Method

1. It is limited in scope. After discovering the formula, it does not provide for sufficient practice in problem solving. The discovery of a formula does not complete the study of the topic.
2. Inductive reasoning is not absolutely conclusive. It establishes a certain degree of probability which increases or decreases in proportion to the number of concrete cases taken.
3. It is laborious and time consuming method.
4. The examples may divert the attention from the rule of formula which is the main thing. Too many examples may obscure the rule.
5. At the advanced stage it is not so useful as too many examples may make teaching dull and boring.

Deductive Method

It is the opposite of inductive method. Here we proceed from general to particular, abstract to concrete and formula to examples. A ready-made formula is given to the students and they are asked to solve relevant problems with its help. The formula is accepted by the learners as well-established truth.

Procedure :

In the very beginning the teacher states the formula of the day. To show the application of the formula to problems, he solves a number of problems, with its help. The student come to observe how the formula can be used or applied. A few more problems may be given for practice. They solve them in the same way as done by the teacher.

For example the teacher may announce the volume of a cuboids = Length x Breadth x Height, and then apply this formula to find the volume of a cupboard 25cm. long, 18cm. wide and 12 cm. high. In another example, he may announce that $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$. Then he may show the application of this formula to the expression $8x^3 - 125y^3$.

Merits of the Deductive Method

1. The method is short and elegant. Being economical of time and labour. It enhances speed, skill and efficiency in problem solving.
2. The books are written on deductive lines, therefore the teacher also prefers to use this method.
3. Being precise it provides a logical cohesion in the mind. It also provides a neat record for references and review.
4. At the practice and revision stage, this method is glorifies memory.
5. As it demands memorisation of formula, therefore it glorifies memory.
6. It combines with the inductive method to remove incompleteness and inadequacy of the latter.

Drawbacks of the Deductive Method

1. It is incomprehensible, often meaningless to the immature learner.
2. There will be undue burden on the learner's mind as he will have to memories a large number of formulae and rules.
3. It places memory at a premium and intelligence at a discount.
4. After forgetting the memorized material the forgetting being very rapid-the student is utterly helpless.
5. The students do not becomes active learners.
6. It is not suitable for thinking, reasoning and exploration.

Conclusion

(Combination of Inductive-Deductive Methods)

The discussion of the procedure, merits and drawbacks of the two methods leads us to conclude that they have to be combined. All new teaching should be started with inductive method and should end in deductive method. Mathematics in the beginning is inductive and its finished form is deductive. Deduction and induction are complementary and not contradictory. The two methods are such good partners that the shortcomings of the one are offset by the other. Any loss to time due to the slow speed of induction can be made up through the time-saving process of deduction. Induction leaves the learners at a point where he cannot stop, the remaining work has to be completed by deduction. The learner understands new things inductively and applies them deductively. Induction gives the lead and deduction follows.

1.4.4 Analytical-Synthetic Methods

These two methods are also complementary in nature. To conclude about them we have to understand them separately.

Analytical-Method

It proceeds from "unknown to known". Analysis means breaking up the problem in hand into parts and then resolving it by connecting the unknown elements with something previously known. The problem is unfolded and the complex situation is broken into manageable items. We start with what is to be found out or what is to be proved and then think of the possibilities which may lead us to the proof. Analysis is often identified with induction. It is the method of discovering the solution of a problem and heuristic attitude is also implicit in it. Evidently analysis is the highest intellectual performance of the mind.

Procedure of the Analytical Method

Ex.1 If $\frac{a}{b}=\frac{c}{d}$ prove that $\frac{ac-3b^2}{b}=\frac{c^2-3bd}{d}$

The analysis will start from the unknown part of the statement

$\frac{ac - 3b^2}{b} = \frac{c^2 - 3bd}{d}$ is to be proved

What should be done to prove it ? (Cross multiplication)

$\frac{ac - 3b^2}{b} = \frac{c^2 - 3bd}{d}$ will be true if acd - $3b^2d$ = bc^2 - $3b^2d$ is true.

What is the next possibility of simplication ?

(Canceling - $3b^2d$ on both the sides) i.e. acd - $3b^2d$ = bc^2 - $3b^2d$ will be true acd = bc^2

Then what next ? (Cancelling 'C' on both the sides)

acd = bc^2 will be true if

ad = bc

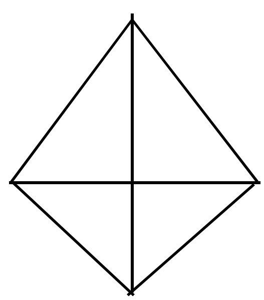
This will true if $\frac{a}{b} = \frac{c}{d}$ which is known and true.

Ex.2 There are two isosceles triangles on opposite sides of the same base, prove that the line joining their vertices bisects the base at right angles.

Given is ABC and BCD are two isosceles triangles on opposite sides of the base BC. We have to prove that AD bisects BC at right angles, or BE = EC and AEC = 90^0.

To prove this we shall have to prove triangles ABE and ACE to be congruent.

But we cannot find enough equal parts to prove the congruency of these triangles.



Now we select another pair of triangles whose congruency will provide equal parts for proving the congruency of previous triangles.

Therefore we prove the congruency of triangle ABD and ACD. There congruency is easily proved.

(S.S.S. = S.S.S.)

Congruency of triangles ABD and ACD leads us to say that

$\angle BAE = \angle CAE$

Now we have adequate number of equal parts to prove the congruency of triangles ABE and ACE.

Thus we can say that the unknown from which we made a start has been connected with something which is known.

Merits of Analytical Method

1. It is a logical method. It does not leave behind any doubts.
2. It resolves the material into simpler components which are easier to handle, to understand and assimilate.
3. It has heuristic spirit in it and strengthens the urge to discover facts.
4. Its steps are developed in a general manner. No clamming of a set pattern is necessitated. Each step has its reason and justification.
5. The student is an active participant in learning. He is throughout faced with such questions as "How to simplify the two sides of an equation ?" and "How to prove the equality of two angles etc. ?"

Drawback of Analytical Method

1. It is a lengthy method. It is difficult to acquire efficiency and speed by this method.
2. It is not a complete method by itself, the problem is solved and completed by syntheses.
3. As soon as the solution is in sight, all of a sudden, the need for further analysis is done.
4. It may not be applicable to all topics equally well.

Synthetic Method

It is the opposite of analytic method. Here we proceed from "known to unknown". In actual practice, it is the complement of analysis. to synthesize means to place together things that are apart. It starts with some thing already known and connects it with the unknown part of the statement. The known parts of information are put together to reach the point where unknown information becomes obvious and true.

Procedure of Synthetic Method

Ex. 1 If $\frac{a}{b}=\frac{c}{d}$ prove that $\frac{ac-3b^2}{b}=\frac{c^2-3bd}{d}$

(We have purposely taken the same example as used in illustrating the analystic method)

$\frac{a}{b}=\frac{c}{d}$ (It is known and hence the starting point)

Substract $\frac{3b}{c}$ on both sides, (But why ? and how he child should remember to A

Subtract $\frac{3b}{c}$ and not any other quantity ?)

$$\frac{a}{b}-\frac{3b}{c}=\frac{c}{d}-\frac{3b}{c}$$

B E C

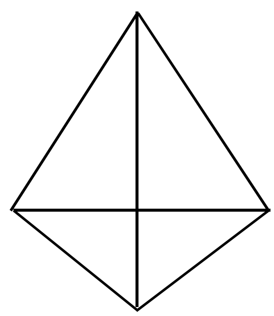
OR $\frac{ac-3b^2}{bc}=\frac{c^2-3bd}{cd}$

OR $\frac{ac-3b^2}{bc}=\frac{c^2-3bd}{cd}$ (cancelling $\frac{1}{c}$ on both sides) D

Hence the identity is proved.

Ex.2 There are two isosceles triangles on opposite sides of the same base. Prove that the line joining their vertices bisects the base at right angles.

We make a beginning from what is given and then try to connect it with the element to be proved.



We take up the newly formed triangles ABD and ACD and prove their congruency. (But why ?)

After proving triangle ABC = triangle ACD, we say that BAE = CAE (But why ?)

With the help of this equality of two angles and other known equalities we prove the congruency of triangles ABC and ACE. (Again this is done without any justification).

From this congruency of triangles, we can show that the unknown part of the statement stand proved.

Merits of Synthetic Method

1. It is the complement and finishing part of every analysis.

2. It is short, an elegant and provides a neat record of work.
3. It glorifies memory as it demands retention of step by step procedure.
4. Students can acquire skill, speed and efficiency with this method.

Drawbacks of Synthetic Method

1. It leaves behind many doubts in the mind of the learner and offers no explanation for them.
2. It does not provide understanding.
3. There is no place for thinking and discovery in this method.
4. Burdens of memory work and homework are likely to become heavy.
5. Without satisfactory explanation for the procedure and questions that arise in synthesis, the pupil becomes helpless when a new problem is set to him.
6. The complete recall of the steps of synthesis may not be possible for the learner.
7. No power is developed in pupils and originality is destroyed.

Conclusion

Analysis combined with synthesis is the prefect method. Discoveries mad by analytic method serves the purpose of clarifying the reason for the steps taken in the construction of a proof and synthetic method puts the whole material in a fine shape. Since analysis is a lengthy method, it needs the help of synthesis for removal of this defect. In the combined method, facts are separated in order to find out how these can be knit together to from one whole. Analysis help in understanding and synthesis help in retaining knowledge. Analysis forms the beginning and synthesis accomplishes the follow up. The teacher would help the students for the analytic form of the solution and synthetic work would be left to the pupils.

A comparative study of the method given below will make them more clear.

	Analytical Method		Synthetic Method
1.	Proceeds from unknown to known.	1.	Proceeds from known to unknown.
2.	It is an exploratory procedure.	2.	It is method for the presentation of discovered facts.
3.	The problem or statement is broken its simpler elements.	3.	It is a process of combining known facts into or statement to get something new.
4.	One starts from what is to be proved and then come back to what is given	4.	Here one starts from what is given and ends with what is to be proved.

5. All steps have full justification and purposes along with a regular sequence.	5. Every step is correct but without good reason. There is no explanation for the steps taken.
6. It is psychological method.	6. It is logical method.
7. The teacher carries the students with him.	7. The teacher is not in touch with the class.
8. It is the process of thinking.	8. It is the product of thought.
9. The method is formational. It is a scientific method leading to spirit of enquiry.	9. This method is information. It is an unscientific method.
10. It is uneconomical in terms of time and labour.	10. It is economical because it is short and concise.
11. Skill, speed and efficiency needed in computation work cannot be properly acquired in this method.	11. It gives essential skill, speed and efficiency to the computation work.
12. It helps the students to become self reliant and self confident.	12. The pupil is just like a man led blind fold to the desired goal.

1.4.5 The Van - Hiele Levels of Geometric thinking - Nature

This theory originated in 1957 by husband and wife team Dina Van Hiele-Geldof and Pierre van Hiele from the Utrecht University in the Netherlands. It helps to illustrate how students learn geometry. The Van Hiele levels have helped shaped curricula throughout the world, including a large influence in the standards of geometry in the US.

The theory has three: the existence of levels, the properties of the levels, and the progress one level to the next level.

Van - Hiele Levels

1) **Visualization**: At this level students use visual perception and non verbal thinking. At this level students do not identify the properties of geometric figures.
2) **Analysis**: At this level students starts analyzing and naming properties of geometric figures. They are not able to see relationship between properties.
3) **Abstraction**: At this level students can distinguish between properties and figures. They are able to create meaningful definitions. They are capable to give simple argument to justify their reasoning.
4) **Deduction:** At this level students can give deductive geometric proofs. They identify which properties are implied by others.

5) **Rigor:** At this level students understand the way how mathematical systems are established. They are able to use all types of proofs.

1.4.6 Kinds of Proof

1.4.6.1 Direct Proof (Proof by construction)

In a constructive proof one attempt to demonstrate P '!Q directly. This method is the simplest and easiest among all kinds of proof. Following are the two steps to a direct proof

i. Assume that P is true.

ii. Use P to show that Q must be true.

Example: If a and b are consecutive integers, then the sum a+b is odd.

Proof: Assume that a and b are consecutive integers. Because a and b are consecutive we know that b= a +1. Thus, the sum a+ b may be rewritten as 2a + 1. Thus, there exists a number k such that a +b = 2k+ 1 so the sum a+ b is odd.

1.4.6.2 Mathematical Induction

The word induction means the method of inferring a general statement from the validity of particular cases. Mathematical induction is a principle by which one can conclude a statement for all positive integers after proving certain related proposition.

The principle of mathematical induction is as follows:

Let P(n) be a statement such that

(i) P(1) is true

(ii) P(r+1) is true whenever P(r) is true

Then P(n) is true for all natural numbers n.

Example: Let P(n) be the statement "$n^2 + n$ is even".

Proof: (i) P(1) is the statement "2 is even". It is true.

(ii) If P(r) is true for some r, then to prove that P(r+1) is true, consider

$(r+1)^2 + (r+1) = r^2 + 2r + 1 + r + 1$

$= r^2 + r + 2(r + 1)$

= an even number + 2(r+1) (because P(r) is true)

= sum of two even numbers

= an even number

Thus P(r+1) is proved to be true, assuming that P(r) is true. Therefore, it follows that P(n) is true for all n by the principle of induction.

1.4.6.3 Proof by Contradiction

The proof by contradiction is grounded in the fact that any proposition must be true or false, but not both true and false at the same time. We arrive at a contradiction when we are able to demonstrate that a statement is both simultaneously true and false, showing that our assumptions are inconsistent.

A general form of proof by contradiction as follows:

i) State that the proof is by contradiction

ii) Assume the statement to be false

ii) Proceed with a direct proof

iii) Come across a contradiction

iv) Now because of the contradiction, it cannot be the case that a statement is false, so it must be true

Example: Prove that for every positive real number x

x/(x+1) < (x+1)/(x+2)

If Rx is the set of all positive real numbers and P(x) denotes the statement

x/(x+1) < (x+1)/(x+2)

then the statement to be proved has the form (for all x € R_x)P(x). Thus, a proof by contradiction begins by assuming the negation, (for all x € R_x) (~P(x)).

Proof: The proof by contradiction, so assume that there exists a real number x such that

x/(x+1) e" (x+1)/(x+2)

Since x is positive, both x+1 and x+2 are positive. Hence, (x+1)(x+2) is positive, so we can multiply both sides of the inequality

x/(x+1) e" (x+1)/(x+2) by (x+1)(x+2) and still preserve the inequality. This gives x(x+2) e" $(x+1)^2$.

Expanding both sides $x^2 + 2x$ e" $x^2 + 2x + 1$.

subtracting $x^2 + 2x$ from both sides leaves 0 e" 1, a contradiction, so we conclude that for every positive real number x,

x/(x+1) < (x+1)/(x+2)

Hence proved

1.4.6.4 Disproof by counter example

A conjecture may be described as a statement that we hope is a theorem. As we know, many theorems are universally quantified statement. Thus it seems reasonable to begin our discussion by investigating how to disprove a universally quantified statement.

Example: Either prove or disprove the following conjecture.

Conjecture: If A, B and C are sets, then A - (B)" C) = (A - B))" (A - C)

Disproof: This conjecture is false because of the following counter example.

Let A = {4, 5, 6}, B= {4, 5} and C = {5, 6}.

Notice that A - (B)" C) = {4, 6} and (A - B))" (A - C) = w,

So A - (B)" C) "" (A - B))" (A - C)

Summary

According to The Van - Hiele it is appropriate to divide the teaching of geometry into different levels. In this lesson we discussed about The Van - Hiele levels and different types of proof.

1.4.7 SUGGESTED QUESTIONS

1. Illustrate and discuss the inductive-deductive methods of teaching Mathematics. What is the justification for their combination ?
2. Discuss the analytical and synthetic method of teaching Mathematics. How will you apply them for the teaching of the proposition on the sum of the angle of triangles ?
3. "No induction is complete without deduction". Discuss the above view-point clearly indicating the limitation's and merits of inductive process.
4. What are the fundamental principles underlying the analytic-synthetic methods in the teaching of mathematics.
5. Discuss the analytic-synthetic method of teaching. Compare them for their similarities and dissimilarities.
6. Discuss good points of inductive-deductive method of teaching Mathematics.
7. What is inductive method ? Illustrate its application in teaching algebra and geometry by one example from each.
8. Discuss the analytic method of teaching with reference to its procedure, merits and limitations.
9. Compare and contrast the analytic and synthetic methods of teaching.
10. Define the term The Van - Hiele Levels of Geometric thinking
11. Describe the Van - Hiele Levels.
12. Explain the proof by mathematical induction with an example.
13. Explain proof by contradiction with an example.

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